D–3151

Sub. Code
31111

DISTANCE EDUCATION

M.Sc DEGREE EXAMINATION, DECEMBER 2019

First Semester

Mathematics

ALGEBRA - I

(CBCS 2018-2019 Academic Year Onwards)

Time : 3 Hours Maximum : 75 Marks

PART A ($10 \times 2 = 20$)

Answer ALL questions.

1. Define Identity mapping.
2. Show that conjugate relations is a equivalence relation on *G*.
3. Show that the intersection of two normal subgroups of a group *G* is also a normal subgroup of *G*.
4. Prove that $a \in Z$ if and only if $N(a) = G$
5. Define normalizer of an element in *G*.
6. Define a maximal ideal of ring with a example.
7. When will you say that a ring is a maximal ideal?
8. State the unique factorization theorem.
9. Let *R* be an Eucilideen domain. Suppose that $a,b,c \in R$, a/bc but $(a,b)=1$.prove that a/c.
10. Find the greatest common divison of the polymials $x^3 - 6x^2 + x + 4$ and $x^5 - 6x + 1$ in $Q[x]$

PART B (5 × 5 = 25)

Answer ALL questions.

11. (a) For any three sets, A,B,C, prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

Or

(b) If a/x and b/x and $(a,b)=1$, prove that $(ab)/x$.

12. (a) If G is a finite group and H is a subgroup of G, then prove that $o(H)$ is a divisor of $o(G)$.

Or

(b) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

13. (a) If $o(G)=p^2$, Where p is prime, prove that G is abelian.

Or

(b) Prove that the homorphism ϕ of R into R^1 is an isomorphism if and only if $I(\phi) = (0)$.

14. (a) Show that the Kernal of a ring homomorphism is a two sided ideal.

Or

(b) Prove that the mapping $\phi : D \to F$ defined by $\phi(a) = [a,1]$ is an isomorphism of D into F.

15. (a) State and prove Einstein criterion therem.

Or

(b) If p is a prime number of the form 4n+1, then prove that $p = a^2 + b^2$ from some integers a and b.

PART C (3 × 10 = 30)

Answer any THREE questions.

16. Prove that any positive integer $a>1$ can be factored in a unique way as $a = p_1^{\alpha_1}.p_2^{\alpha_2} \dots p_t^{\alpha t}$, where $p_1 > p_2 > \dots > p_t$ and prime numbers and where each $\alpha_i > 0$.

17. State and prove Cayle's therem

18. Prove that every integral domain can be imbedded in a field.

19. Show that a commutative rig R with unit elements is a field if and only if the only ideals of R are{o} and R itself.

20. Prove that the ring of Gaussian integers is an Euclidean ring.

D–3152

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2019.

First Semester

ANALYSIS — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. In a metric space, every convergence sequence is bounded.

2. Balls are convex. Justify.

3. Define connected set. Give an example.

4. If a and b are real, show that $(a, b) = a + bi$.

5. Define perfect set. Give an example.

6. Let $S_n = \frac{(-1)^n}{1 + \frac{1}{n}}$. Find $\lim_{n \to \infty} \sup S_n$.

7. Show that every open ball in $\mathbb{R}^n$ is convex.

8. Let f be defined on $[a, b]$. If f is continuous at $x \in [a, b]$, then f is differentiable at x. Justify.

9. State the generalized mean value theorem.

10. State the intermediate value theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that every non-empty set of natural numbers has a smallest member.

Or

(b) Prove that a non degenerate interval of real numbers is uncountable.

12. (a) Prove that the subsequential limits of a sequence $\{p_n\}$ in a matrix space X form a closed subset of X.

Or

(b) Prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

13. (a) If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, then prove that $f(E)$ is connected.

Or

(b) Let f be a continuous real valued function on a metric space X. Let $z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Prove that $z(f)$ is closed.

14. (a) Let f be a real differentiable function on $[a, b]$ and let $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

Or

(b) Let f be defined for all real x, and $|f(x) - f(y)| \le (x - y)^2$ for all real x and y. Prove that f is constant.

15. (a) Suppose f maps an open set $E \subset R^n$ into R^m. If $f \in \mathscr{C}'(E)$ then prove that the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \le i \le m$, $1 \le j \le n$.

Or

(b) If f is a $\mathscr{C}'$-mapping of an open set $E \subset R^n$ into R^n and if $f'(x)$ is invertible for every $x \in X$, then prove that $f(w)$ is an open subset of R^n for every open set $W \subset E$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that for every real $x > 0$ and every integer $n > 0$, there is one and only one real y such that $y^n = x$.

17. Let p be a non-empty perfect set in R^k. Prove that p is uncountable.

18. Prove that $\Sigma \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.

19. State and prove Bolzano Weierstrass theorem.

20. State and prove Taylors theorem.

D–3153

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2019.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State the existence theorem for initial value problem.

2. Define Wronskian of $\varphi_1, \varphi_2, \ldots.., \varphi_n$.

3. Find the singular points of $x^2y'' + (x^2 + x)\,y' - y = 0$.

4. Find two linearly independent solutions of $y'' + \frac{1}{4x^2}\,y = 0, x > 0$.

5. Solve $y'' - 4y' + 5y = 0$.

6. Find all solutions of $y'' - 4y' + 5y = 3e^{-x}$.

7. State the second order Euler equation.

8. Compute the indicial polynomial and its roots for the differential equation $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$.

9. State the exterion Neumann problem.

10. Find the value of $P_n(1)$ and $P_n(-1)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If φ_1 and φ_2 are two solutions of $L(y)=0$ on an interval I containing a point x_0, then prove that $W(\varphi_1\ \varphi_2)(x)=e^{-a_1(x-x_0)}\,W(\varphi_1,\varphi_2)(x_0)$.

Or

(b) Verify that $\varphi_1(x)=x^3$, $x>0$ satisfies $x^2y''-7xy'+15y=0$. Find the second independent solution.

12. (a) Find all solutions of $x^2y''+2xy'-6y=0$ for $x>0$.

Or

(b) Prove that $(x^{-\alpha}J_\alpha)'=-x^{-\alpha}\,J_{\alpha+1}^{(x)}$.

13. (a) Compute the indicial polynomial of $x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y=0$.

Or

(b) By computing appropriate Lipschitz constants, show that $f(x,y)=x^2\cos^2 y+y\sin^2 x$, on $S:|x|\le 1$, $|y|<\infty$ satisfy Lipschitz conditions.

14. (a) Verify whether the equation $2xy\,dx+(x^2+3y^2)\,dy=0$ is exact or not, if exact solve.

Or

(b) Find the integrating factor of the equation $(2xy^3+2)\,dx+3xy^2dy=0$ and hence solve it.

15. (a) Show that the coefficient of x^n in $P_n(x)$ is $\frac{(2n)!}{2^n (n!)^2}$.

Or

(b) Is it possible to reduce the Neumann problem to Dirichlet problem? Justify.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all solutions of

(a) $y'' + 9y = \sin 3x$

(b) $4y'' - y = e^x$.

17. Derive Bessel's function of first kind of order α, $J_\alpha(x)$.

18. If $\varphi_1, \varphi_2, ...\varphi_n$ are n solutions of $L(y) = 0$ on an interval I, they are linearly independent there if and only if $W(\varphi_1, ..., \varphi_n)(x) \neq 0$ for all x in I - Prove.

19. Let f be a continuous vector-valued function defined on $S: |x - x_0| \leq \alpha$, $|y| < \infty$, $\alpha > 0$ and satisfy a Lipschitz condition, prove that the successive approximation $\{\varphi_k\}$ for the problem $y' = f(x, y)$, $y(x_0) = y_0$, $(|y_0| < \infty)$, exist on $|x - x_0| \leq \alpha$ and converge there to be a solution φ of this problem.

20. Show that all real-valued solutions of the equation $y'' + \sin y = b(x)$ where b is continuous for $-\infty < x < \infty$, exist for all real x.

D–3154

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2019.

First Semester

Mathematics

TOPOLOGY – I

(CBCS – 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define subjective function.

2. State the axiom of choice.

3. Define countably infinite set, give an example.

4. Define product topology with an example.

5. Is the rationals Q connected? Justify.

6. Define metric topology.

7. When will you say a topological space is metrizable?

8. Define Lebergue number.

9. Define completely regular space.

10. Is the uniform topology $\mathbb{R}^{w}$ satisfies the first countablity axiom? Justify.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) State and prove strong induction principle.

Or

(b) If C is an infinite subset of Z_+, then prove that C is countably infinite.

12. (a) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

Or

(b) State and prove pasting lemma.

13. (a) State and prove uniform limit theorem.

Or

(b) Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by $f(a) = (f_\alpha(a))_{\varepsilon \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let $\prod X_\alpha$ has product topology. Prove that there is a function f is continuous if and only if each function f_α is continuous.

14. (a) Prove that every compact subspace of a Hauasdorff space is closed.

Or

(b) Prove that a space X is locally connected if and only if for every open set U of x, each component of U is open in X.

15. (a) Prove that every metrizable space is normal.

Or

(b) Prove that a subspace of a completely regular space is completely regular.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If A is finite then prove that there is no bijection of A with proper subset of itself.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metrical and the square metric S are the same as the product topology on $\mathbb{R}^n$.

18. If X is a topological space, each path component of X lies in a component of X. If X is locally path connected then prove that the components and the path components of X are the same.

19. State and prove tube lemma.

20. State and prove Urysohn metrization theorem.

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D–3155

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2019.

Second Semester

ALGEBRA - II

(CBCS 2018-19 Academic year onwards)

Time : Three hours — Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define vector space homomorphism.

2. In a vector space show that $\alpha\ (V - W) = \alpha V - \alpha W$.

3. Prove that the annihilator of a subspace W is a subspace of $\hat{V}$.

4. Prove that $L\ (S)$ is a subspace of V.

5. If $S, T \in Hom\ (V, W)$ and $V_i S = V_i T$ for elements V_i of a basis of V, prove that $S = T$.

6. Define algebraic extension over a field F.

7. Define fixed field of a group of automorphisms on field. Find the fixed field of the trivial group in $Q\sqrt[3]{2}$.

8. Express the polynomial $x_1^2 + x_2^2 + x_3^2$ in the elementary symmetric functions in x_1, x_2, x_3.

9. Define self-adjoint and Hermitian adjoint of $T \in A(V)$.

10. If V is a finite dimensional over F and if $T \in A(V)$ is invertible, then show that T^{-1} is a polynomial expression in T over F.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If S, T are subsets of V, then prove that

(i) $S \subset T \Rightarrow L(S) \subset L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$

(iii) $L(L(S)) = L(S)$.

Or

(b) Prove that if T is a homomorphism of U onto V with Kernal W, then W is isomorphic to U/W.

12. (a) If V is a finite dimensional and $v \neq 0 \in V$, then show that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

Or

(b) Define orthogonal complement of a subspace W and prove that if is a subspace of V.

13. (a) If a, b in K are algebraic over F, then prove that $a \pm b$, $a \cdot b$, a/b (provided $b \neq 0$) are all algebraic over F.

Or

(b) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

14. (a) If K is the field of complex numbers and F is the field of real numbers. Compute $G(K, F)$.

Or

(b) Prove that $G(K, F)$ is a subgroup of the group of all automorphisms of K.

15. (a) Prove that the characteristic vectors corresponding to district characteristic roots of $T \in A(V)$ are linearly independent over F.

Or

(b) If $T \in A(V)$ the prove that

(i) $T^* \in A(V)$

(ii) $(T^*)^* = T$

(iii) $(S+T)^* = S^* + T^*$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If V is a vector space over F and if W is a subspace of V then prove that $\frac{V}{W}$ is a vector space over F under the suitable operations.

17. Establish Gram-Schmidt's orthogonalization process.

18. Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

19. State and prove the fundamental theorem of Galois theory.

20. Prove that the linear transformation T on V is unitary if and only if it takes on orthonormal basis of V into an orthonormal basis of V.

D–3156

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2019.

Second Semester

Mathematics

ANALYSIS – II

(CBCS – 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Riemann – Stieltjes integrable functions.

2. Find $U(p, f, \alpha)$ where $P = \{0, 0.2, 0.4, 0.6, 0.8, 1, 1.5, 2\}$ $\leq [0,2]$ and $f;\alpha : [1,2] \to IR$ define by $f(x) = [x]$ and $\alpha(x) = 2$.

3. Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2x}$. For what values of x does the series converges absolutely?

4. Is $\{f_n(x)\}$ converge uniformly on $(0, 1)$? where $f_n|n| = \frac{1}{1+nx}$ Justify.

5. Write the formula for the radius convergence.

6. Show that the measurable set E is periodic.

7. Prove that m^* is translation invariant.

8. Define Borel set.

9. If f is non negative simple measurable function and $c > 0$, show that $\int_E cf = c\int_E f$.

10. Give an example of Lebesgue integrable function which is not Riemann integrable.

PART B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) If f is continuous on $[a,b]$ then prove that $f \in R(\alpha)$ $m[a,b]$.

Or

(b) If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on $[a, b]$ then show that $f_1 + f_2$, $cf_1 \in R(\alpha)$ and $\int_a^b (f_1 + f_2)d\alpha = \int_a^b f_1 dx + \int_a^b f_2 dx$, $\int_a^b cf, d\alpha = c\int_a^b f_1 d\alpha$.

12. (a) State and prove Weierstrass M – test.

Or

(b) Show that $\mathcal{C}(x)$ is a complete metric space.

13. (a) Let $\{a_{ij}\}$ be double sequence. Suppose $\sum_{j=1}^{\infty}|a_{ij}| = b_i$ and $\sum b_i$ converges. Then show that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

Or

(b) If f is a positive function on $(0, \infty)$ such that

(i) $f(x+1) = x(f(x)$,

(ii) $f(1) = 1$

(iii) $\log f$ is convex, then show that $f(x) = \Gamma(x)$

14. (a) If E_1 and E_2 are measurable, then show that $E_1 v E_2$ is measurable.

Or

(b) If f is a measurable function and $f = g$ a.e then show that g is measurable.

15. (a) State and prove bounded convergence theorem.

Or

(b) Let f be a non – negative function which is integrable over a set E. Then show that for any $\varepsilon > 0$, there is a $\delta > 0$ such that $\int_A f < \varepsilon$. For every $A \subset E$ with $mA < \delta$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let α be a monotonically increasing function and $\alpha^1 \in \mathfrak{R}$ on [a, b]. let *f* be a bounded real function on [a, b]. Then that $f \in \mathfrak{R}(\alpha)$ if and only if $f\alpha' \in \mathfrak{R}$, Further $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

17. State and prove Stone – Weierstrass theorem.

18. Prove that every non – constant polynomial with complex co – efficient has a complex root.

19. Prove that the outer measure of an internal is its length.

20. Let *f* be bounded on a measurable set E with $mE < \infty$. Then show that *f* is measurable if and only if $\inf_{f \le \Psi} \int_E \Psi(x)dx = \sup_{f \ge \varphi} \int_E \varphi(x)dx$ for all simple function φ and Ψ.

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D–3157

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2019.

Second Semester

Mathematics

TOPOLOGY – II

(CBCS – 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define locally metrizable space.

2. State the countable intersection property.

3. Show that the rationals Q are not locally compact.

4. Define complete metric space.

5. State locally finite property in topological space.

6. Define compactly generated space.

7. Show that the real line R is neither bounder not totally bounded under the metric $d(x,y)=|x-y|$.

8. State totally bounded metric space.

9. Let $A \subset X$; let $f : A \to Z$ be a continuous map of A into the Hausdorff space Z. Show that there is atmost one extension of f to a continuous function $g : \overline{A} \to z$.

10. Define Baire space with an example.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Let X be locally compact Hausdorff space ; let A be a subspace of X. If A is closed in X or open in X, then prove that A is locally compact.

Or

(b) Let X be a locally compact Handsorff space which is not compact. Let Y be the one – point compactification of X.

Prove that

(i) Y is a compact hausdorff space

(ii) X is a subspace of Y.

12. (a) Show that any completely regular space X can be imbeded in $[0,1]^J$.

Or

(b) Prove that every Paracompact Hausdorff space X is normal.

13. (a) Show that the collection of intervals $A = \{(n, n+2) / n \in Z\}$ is locally finite in the topological space $\mathbb{R}$.

Or

(b) Prove that every metrizable space is Paracompact.

14. (a) Prove that Euclidean space $\mathbb{R}^k$ is complete in either of its usual metrics, the euclidean metric d or the square metric S.

Or

(b) If X is locally compact or X satisfies the first countability axiom, then show that X is compactly generated.

15. (a) Prove that any open subspace Y of a Baire space X is itself a Baire space.

Or

(b) Show that any discrete space has dimension O.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tietze extension theorem.

17. Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering $\mathcal{C}$ of X refining A that is countably locally finite.

18. State and prove Nagata – Smirnov metrization theorem.

19. Let $I = [0, 1]$. Prove that there exists a continuous map $f : I \rightarrow I^2$ whose image fills up the entire square I^2.

20. State and prove the classical version of Ascoli's theorem.

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D–3158

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,

DECEMBER 2019.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define oblique trajectory.

2. Determine the integrability of the equation.

   $ydx + xdy + 2zdz = 0$.

3. Define the degree of partial differential equation.

4. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x+a)(y+b)$.

5. Define complete integral.

6. If $u = f(x+iy) + g(x-iy)$, where f and g are arbitrary functions, show that $u_{xx} + u_{yy} = 0$.

7. Solve $(D^2 - D^1)z = 0$.

8. State the exterior churchill problem.

9. What is the necessary condition for the existence of the solution of interior Neumann problem?

10. Classify the one dimensional wave equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}.$$

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Determine the integrability of $yzdx + 2xzdy - 3xydz = 0$ and find the solution.

Or

(b) Solve the equation

$$a^2y^2z^2dx + b^2z^2x^2dy + c^2x^2y^2dz = 0.$$

12. (a) Eliminate the arbitrary function *f* from $z = f\left(\frac{xy}{z}\right)$.

Or

(b) Find the general integral of the partial differential equation $z(xp - yq) = y^2 - x^2$.

13. (a) Solve $x(y^2 + z^2)p + y(z^2 + x^2)q = z(y^2 - x^2)$.

Or

(b) Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.

14. (a) Find the complete integral of $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$.

Or

(b) Find a complete integral of $p^2x + q^2y = z$ by Jacobi's method.

15. (a) Find a particular integral of the equation $(D^2 - D^1)z = 2y - x^2$.

Or

(b) Find the D'Alembert's solution of one dimensional wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the orthogonal trajectories on the curve $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$.

17. Find the general solution of $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$.

18. Find the equation of the integral surface of the partial differential equation $2y(z-3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0$ and $x^2 + y^2 = 2x$.

19. Show that the equation $xpq + yq^2 = 1$ has complete integrals

(a) $(z+b)^2 = 4(ax+y)$

(b) $Kx(x+h) = K^2 y + x^2$.

20. The points of trisection of a string are pulled aside through a distance E on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid point of the string always remains at rest.

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D–3159

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2019.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define an osculating plane.

2. Define a circular helix.

3. Define the anchor ring.

4. What is meant by direction coefficients?

5. Define Geodesics.

6. Write the canonical equations for geodesics.

7. Write the formula for geodesic curvature kg.

8. Write short note on Gaussian curvature.

9. Define mean curvature.

10. When will you say the conic is Dupin's indicatrix?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Determine the intrinsic equations of the curve $\bar{r} = \left[ae^u \cos u, ae^u \sin u, be^u\right]$.

Or

(b) With usual notations, prove that $[\bar{r}', \bar{r}'', \bar{r}'''] = k^2\tau$.

12. (a) Show that on the right helicoid, the family of curves orthogonal to the curves $u.\cos v =$ constant is the family $\left(u^2 + v^2\right)\sin^2 v =$ constant.

Or

(b) Show that the metric is invariant under a parameter transformation.

13. (a) Enumerate the normal property of geodesics.

Or

(b) Prove that the curves of the family $\dfrac{v^3}{u^2} =$ constant are geodesics on a surface with metric $v^2du^2 - 2uv\,du\,dv + 2u^2\,dv^2$ $(u > 0, v > 0)$.

14. (a) Derive the Liouville's formula for Kg.

Or

(b) Prove that the components λ, μ of the geodesic curvature vector are given by the following formula with s as parameter,

$$\lambda = \frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial v'} = \frac{-1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial v'}$$

$$\mu = \frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial u'} = \frac{-1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial u'}$$

15. (a) Enumerate the polar developable.

Or

(b) Prove that the edge of regression of the rectifying developable has equation $\overline{R} = \overline{r} + k\left(\frac{\tau\,\overline{t} + k\overline{b}}{k'\tau + k\tau'}\right)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Serret-Frenet Formulae.

17. Find the surface of revolution which is isometric with a regions of the right helicoid.

18. Find the geodesics on a surface of revolution.

19. State and prove Gauss-Bonnet theorem.

20. Prove that a necessary and sufficient condition for a surfaces to be a developable is that its Gaussian curvature shall be zero.

D–3160

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2019.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define network, directed network.

2. Define Tree, Spanning tree.

3. What is Optimality and feasible condition of revised simplex method?

4. Write down matrix form of simple method.

5. Define Two-person zero sum game.

6. Define global maximum gloabl minimum.

7. Write down the Floyd's algorithm.

8. Define strategies of a game and value of the game.

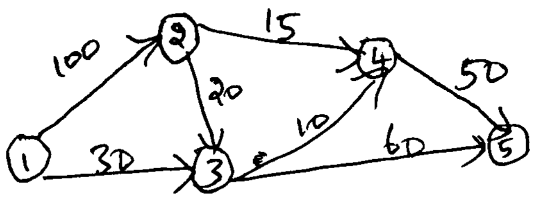
9. What is gradient method?

10. What is separable programming?

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) The network below gives the permissible routes and their lengths in miles between city 1 and four other nodes (cities). Determine the shortest routes between city 1 and each of the remaining cities.



Or

(b) Write down the Maximal Flow Algorithm.

12. (a) What are the total and free floats of a critical activity?

Or

(b) Determine all the basic feasible solution of the following system of equation.

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

13. (a) Solve by revised simplex Algorithm for the following LP.

Maximize $Z = 6x_1 - 2x_2 + 3x_3$

Subject to
$$2x_1 - x_2 + 2x_3 \leq 2$$
$$x_1 + 4x_3 \leq 4$$
$$x_1\ x_2, x_3 \geq 0.$$

Or

(b) Explain bounded variable Algorithm.

14. (a) Determine the strategies that the saddle point and value of the game for the following payoff matrix.

	B_1	B_2	B_3	B_4
A_1	9	6	2	8
A_2	8	9	4	5
A_3	7	5	2	5

Or

(b) Explain the solution of mixed strategy.

15. (a) Determine the extreme point of the following function.

$$f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$$

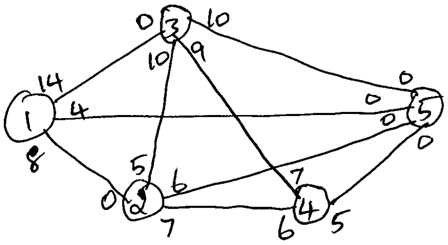
Or

(b) Explain Jacobian method.

PART C — (3 × 10 = 30 marks)

Answer any THREE of the following.

16. Determine the maximal flow for the network given below.



17. Solve the following LP by bounded algorithm method.

Maximize $Z = x_1 + 2x_2$ Subject to

$-x_1 + 2x_2 \geq 0$

$3x_1 + 2x_2 \leq 10$

$-x_1 + x_2 \leq 1$

$1 \leq x_1 \leq 3, 0 \leq x_2 \leq 1.$

18. Solve by Newton-Raphson method of $f(x) = 4x^4 - x^2 + 5$.

19. Solve the following LP by Jacobian method.

Maximize $f(x) = 5x_1 + x_2$ subject to

$$g_1(x) = x_1 + 2x_2 + x_3 - 6 = 0$$
$$g_2(x) = 3x_1 + x_2 + x_4 - 9 = 0$$
$$x_1, x_2, x_3, x_4 > 0.$$

20. Solve the following problem by gradient method.

Maximize $f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$.

D–3161

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2019.

Third Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS – 2018 – 2019 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. If $(a,b)=(a,c)=1$ prove that $(a,bc)=1$.

2. Define the Mobius function μ and write down the values of $\mu(n)$, for $1 \le n \le 10$.

3. When will you say the functions are divisor functions?

4. If *f* is multiplicative, then prove that $f(1)=1$.

5. State Legeudre's identity.

6. What are the solutions of the congruence $x^2 \equiv 1 (\bmod 8)$?

7. State the Wilson's theorem.

8. Determine whether -104 is a quadratic residue or non residue of the prime 997.

9. State Wolstenholme's theorem.

10. Define reduced fraction

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

Or

(b) Prove that if $n \geq 1$, $\log n \sum_{d/n} \wedge (d)$.

12. (a) If both g and $f*g$ are multiplicative, prove that f is also multiplicative.

Or

(b) State and prove Selberg's identity.

13. (a) Prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + o\left(\frac{1}{x}\right)$, if $x \geq 1$.

Or

(b) Prove that the numbers which are congnient mod m have the same g.c.d with us

14. (a) State and prove Chinese Remainder theorem.

Or

(b) State and prove Euler's criterion.

15. (a) For every add prime p, prove that $(z/p) = (-1)^{(p^2-1)/8} = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod 8 \\ -1 & \text{if } p \equiv \pm 1 \pmod 8 \end{cases}$.

Or

(b) Show that the Legendre's symbol (u/p) is a completely multiplicative function of n

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) State and prove Euclidean algorithm

(b) Show that the infinite series $\sum_{n=1}^{\infty} 1/p_n$ diverges.

17. (a) With usual notations, prove that $\dfrac{n}{\phi(n)} = \sum_{d/x} \dfrac{\mu^2(d)}{\phi(d)}$

(b) State and prove generalized inversion formula.

18. Prove that the set of lattice points visible from the origin has density $6/\pi^2$.

19. State and prove Gauss lemma.

20. Prove that the diophantine equation $y^2 = x^3 + k$ has no solutions if k has the form $k = (4n-1)^3 - 4m^2$ where m and n are integers such that no prime $p \equiv -1 \pmod 4$ divides m.

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D-3162

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2019.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 19 Academic year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Markov Chain.

2. Define Stochastic graph.

3. What do you meant by residual time?

4. Define diffusion process.

5. Is the Wierner process, a covariance stationary? Justify.

6. What is meant by traffic intensity?

7. Define Markov Renewal branching process.

8. Define Queuing time.

9. Write the Erlaug's formula.

10. Write the blocking formula.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) The Kolmogorov equations for $i = 0,1$ are $P'_{i0}(t) = -aP'_{i0}(t) + bP'_{i1}(t)$; $P'_{i1}(t) = aP'_{i0}(t) - bP'_{i1}(t)$. Find the transition probability $p_{ij}(t)$.

Or

(b) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution $P_r(X_0 = i) = \frac{1}{3}$, $i = 0,1,2$. Find $P_r(X_2 = 1, X_0 = 0)$

12. (a) Let $\{X(t), t \geq 0\}$ be a Wierner process with $\mu = 0$ and $X(0) = 0$. Find the distribution of T_{a+b}, for $0 < a < a + b$.

Or

(b) Let $X(t)$ be the displacement process corresponding to the velocity process $u(t)$. Show that in equilibrium $E[X(t) - X(0)] = 0$ and $\text{Var}(X(t) - X(o)) = \sigma^2 (\beta t - 1 + e^{-\beta t}) / \beta^3$.

13. (a) Prove that $E\{X_{n+r} \mid X_n\} = X_n m^r$, for $r, n = 0,1,2 \ldots$

Or

(b) For a Galton – Watson process with $m = 1$ and $\sigma^2 < \infty$, prove that $\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{1-P_n(s)} - \frac{1}{1-s}\right\} \to \frac{\sigma^2}{2}$ uniformly in $0 \le s < 1$.

14. (a) Obtain the expected waiting time in the system for M/M/S model.

Or

(b) Find the waiting time density and expected waiting time for M/M (1, b)/1 model.

15. (a) Let $\{X(t), t \ge 0\}$ be a Wierner process with $X(0) = 0$, $\mu = 0$ and let $M(t) = \max_{0 \le s \le t} X(s)$. Find $P_r(M(t) \ge a)$.

Or

(b) Derive the Fokker – Planck equation.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. For the pure birth process, show that the interval T_k between the k^{th} and $(k+1)^{\text{th}}$ birth has an exponential distribution with parameter λ_k.

17. Prove that $P_n(s) = P_{n-1}(P(s)) = P(P_{n-1}(s))$.

18. For a Galton – Watson process, show that the expected number of total progeny is given by

$$E(Y_n) = \begin{cases} \left(1 - m^{n+1}\right)/1 - m, \ if \ m \ne 1 \\ n+1, \ if \ m = 1 \\ \dfrac{1}{1-m}, \ if \ m < 1 \ and \text{ as n} \to \infty \end{cases}$$

Show that Var $(Y_n) = (2n+1)(n+1)n\sigma^2/6$, if $m = 1$.

19. Machine interference with m machines and s repairman,

(a) Show that the mean arrival rate λ' is given by $\lambda' = \sum_{n=0}^{s} (m-n)\lambda P_n = \lambda(m-L)$ and that $W = \frac{L}{\lambda'} = \frac{L}{\lambda(m-L)}$.

(b) Denote by X, the expected number of machines in working order. Show that, when s is close to m, then $X \cong \frac{m}{1+\frac{\lambda}{\mu}}$ and when s is small, then $X \cong \frac{s}{\left(\frac{\lambda}{\mu}\right)}$.

20. Derive the Chapman – Kolmogorov equation.

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D–7514

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

First Semester

Mathematics

ALGEBRA – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define one to one and onto functions.

2. Define abelian group. Give an example.

3. Define normal subgroup of a group G.

4. Show that $a \in Z$ if and only if $N(a) = G$.

5. State the Pigeonhole principle.

6. Show that (3) is a prime ideal of Z.

7. Define Kernal of a homomorphism of a ring.

8. Define principal ideal ring.

9. Show that Z is an Euclidean domain where $d(a) = |a|$.

10. State the unique factorization theorem.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) For any three sets A, B, C, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Or

(b) Let $\theta : G \to H$ be onto group homomorphism with kernal K. Prove that G/K is isomorphic to H

12. (a) If n is a positive integrer and a is relatively prime to n, prove that $a^{\phi(n)} \equiv 1 \bmod n$

Or

(b) State and prove second part of sylow's theorem.

13. (a) If ϕ is a homomorphism of R into R' with kernal $I(Q)$, then prove that

(i) $I(\phi)$ is a subgroup of R under addition.

(ii) If $a \in I(\phi)$ and $\gamma \in R$ then both ar and ra are in $I(\phi)$

Or

(b) Prove that a finite integral domain is a field.

14. (a) Let R be an Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

Or

(b) Prove that the kernal of a ring homomorphism is a two sided ideal.

15. (a) Let $f(x), g(x)$ be two non zero elements of $F[x]$. Prove that $\deg(f(x).g(x)) = \deg f(x) + \deg\ g(x)$

Or

(b) Let R is an integral domain. Prove that $R[x]$ is also an integral domain.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Lagrange's theorem.

17. State and prove Cayley's theorem for abelian groups.

18. Prove that every integral domain can be imbedded in a field.

19. If v is an ideal of a ring R, then prove that R/V is a ring and is a homomorphic image of R.

20. State and prove the Eisenstein Criterian theorem.

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D–7515

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define ordered set. Give an example.

2. Define the cantor set. Give an example.

3. Give an example to show that disjoint set need not be separated.

4. When will you say a metric space is complete?

5. Define power series. Also find the radius of convergence of the power series $\sum n^3 z^n$.

6. Define bounded function.

7. State the ratio test.

8. If $\sum a_n$ converges absolutely, then prove that $\sum a_n$ converges.

9. If $f(x) = x^n;\ x \in R$, then prove that $f'(x) = n\ x^{n-1}$.

10. State the L'-Hospital's rule.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $a_1, ..., a_n$ and $b_1, ..., b_n$ are complex numbers, then prove that $\left|\sum_{j=1}^{n} a_j b_j\right|^2 \leq \sum_{j=1}^{n} |a_j|^2 \ \sum_{j=1}^{n} |b_j|^2$.

Or

(b) State and prove Weierstress theorem.

12. (a) Define e, prove that e is irrational.

Or

(b) Prove the following :

(i) If $p > 0$, then $\lim_{n \to \infty} \frac{1}{n^p} = 0$.

(ii) If $p > 0$, then $\lim_{n \to \infty} \sqrt[n]{p} = 1$.

13. (a) State and prove Merten's theorem.

Or

(b) If $\sum a_n$ is a series of complex numbers which converges absolutely, then prove that every rearrangement of $\sum a_n$ converges, and they all converges to the same sum.

14. (a) Show that monotonic functions have no discontinuities of the second kind.

Or

(b) Suppose f is a continuous mapping of a compact metric space x into a metric space y. Prove that $f(x)$ is compact.

15. (a) State and prove Taylor's theorem.

Or

(b) Examine the differentiability of the function f given by

$$f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$$

Also verify that f is continuous at 0.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. For every real $x>0$ and every integer $n>0$, prove that there is one and only one real y such that $y^n = x$.

17. Suppose $Y\subset X$. Prove that a subset E of Y is open relative to Y if and only if $E=Y\cap G$ for some open subset G of X.

18. Let $\sum a_n$ be a series of real numbers which converges, but not absolutely. Suppose $-\infty\leq\alpha<\beta\leq\infty$. Prove that there exists a rearrangement $\sum a_n'$ with partial sums s_n' such that $\lim\limits_{n\to\infty}\inf s_n'=\alpha$ and $\lim\limits_{n\to\infty}\sup s_n'=\beta$.

19. Prove that a continuous image of a connected subset of a metric space x is connected.

20. State and prove the implicit function theorem.

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D–7516

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State the uniqueness theorem.

2. Define Wronskian.

3. Find the real valued solution of $y''+y=0$.

4. Prove that $P_n(-1)=(-1)^n$.

5. Compute the indicial polynomial and its roots for the differential equation $x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y=0$.

6. Define regular singular point.

7. State the second order Euler equation.

8. Compute the first four successive approximations $\varphi_0, \varphi_1, \varphi_2, \varphi_3$ for the equation $y'=y''$, $y(0)=0$.

9. Solve $y' = 3y^{2/3}$.

10. State the local existence theorem for the initial value problem $y' = f(x, y), y(x_0) = y_0$ on z.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve $y'' - 2y' - 3y = 0$, $y(0) = 0, y'(0) = 1$.

Or

(b) Find all solutions of $y'' + y = 2\sin x \sin 2x$.

12. (a) Let $L(y) = 0$ be an n^{th} order differential equation on an interval I. Show that there exist n linearly independent solution of $L(y) = 0$ on I.

Or

(b) Show that the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n (n!)^2}$.

13. (a) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\varphi_1(x) = x^2$. Find the all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

Or

(b) Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\sqrt{(1/2)}} \sin x$ and

$x^{1/2} J_{-1/2}(x) = \dfrac{\sqrt{2}}{\sqrt{(1/2)}} \cos x$.

14. (a) Let f be continuous and satisfy Lipschitz condition R. If φ and ψ are two solutions of $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I containing x_0, then prove that $\varphi(x) = \psi(x)$ for all x in I.

Or

(b) Verify whether the equation $2xy\,dx + \left(x^2 + 3y^2\right)dy = 0$ is exact or not, if exact solve it.

15. (a) Prove that between any two positive zeros of J_0 there is a zero of J_1.

Or

(b) Consider $y'_1 = 3y_1 + xy_3$, $y'_2 = y_2 + x^3 y_3$, $y'_3 = 2xy_1 - y_2 + e^x y_3$. Show that every initial value problem for this system has a unique solution which exists for all real x.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Find the solution φ of the initial value problem $y''' + y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.

17. With the usual notations, prove that

$$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{x}{2}\right)^{2m}.$$

18. Find two linear independent solution of the legendre equation $\left(1 - x^2\right)y'' - 2xy' + \alpha(\alpha + 1)y = 0$.

19. Derive Bessel functions of zero order of the first kind $J_0(x)$.

20. Let f be a real-valued continuous function on the strip $S : |x - x_0| \leq a$, $|y| < \infty$, $a > 0$ and f satisfies on S a Lipschitz condition with constant $K > 0$. Prove that the successive approximations $\{\varphi_k\}$ for the problem $y' = f(x, y)$, $y(x_0) = y_0$ exist on the entire interval $|x - x_0| \leq a$ and converge there to a solution φ of $y' = f(x, y)$, $y(x_0) = y_0$.

D–7517

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define onto function. Give an example.

2. Define dictionary order relation.

3. State the maximum principle theorem.

4. Is the subset $[a,b]$ or $\mathbb{R}$ closed? Justify your answer.

5. What are the continuous maps $f : \mathbb{R} \to \mathbb{R}_l$?

6. Define a linear continuum.

7. Define locally path connected.

8. Define first countable space.

9. Define normal space.

10. What is meant by completely regular space?

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) State and prove well-ordering property.

Or

(b) Prove that a finite product of countable set is countable.

12. (a) Let Y be a subspace of X. Prove that a set A is closed in Y if and only if equals the intersection of a closed set of X with Y.

Or

(b) State and prove the sequence lemma.

13. (a) State and prove the intermediate value theorem.

Or

(b) Let A be a connected subspace of X, If $A \subset B \subset \overline{B}$, then prove that B is also connected.

14. (a) Show that every closed subspace of a compact space is compact.

Or

(b) Prove that a subspace of a completely regular space is completely regular.

15. (a) Prove that every metrizable space with a countable dense subset has a countable basis.

Or

(b) Prove that a product of regular space is regular.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Define countable and uncountable set. Prove that the set $\rho\ (\mathbb{Z}_+)$ of all subsets of $\mathbb{Z}_+$ is uncountable.

17. Prove that the collection

$S = \{\pi_1^{-1}(\mho)/\mho \text{ is open in } X\} \cup \{\pi_2^{-1}(V)/V \text{ is open in } Y\}$ is a subbasis for the product topology on $X \times Y$.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L.

19. Let X be a simply ordered set having least upper bound property. Prove that in the order topology, each closed interval in X is compact.

20. State and prove Urysohn lemma.

D–7518

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

Second Semester

Mathematics

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. When will you say a vector space is a homomorphism?

2. Define the linear span.

3. Define the fixed field of a group automorphism.

4. What is meant by the Galois group of $f(x)$?

5. Define an invertible element.

6. Define a characteristic vector of T.

7. Define hermition and skew hermition.

8. Define companion matrix of $f(x)$.

9. Write the rational canonical form of the matrix of T in $A_F(V)$.

10. Define the range of T.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If V is the internal direct sum of $v_1, v_2, \ldots\ldots, v_n$, then prove that V is isomorphic to the external direct sum of $v_1, v_2, \ldots\ldots, v_n$.

Or

(b) Prove that $A(W)$ is a subspace of V.

12. (a) State and prove the Bessel's inequality.

Or

(b) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

13. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) If K is a finite extension of F, prove that $G(K,F)$ is a finite group and its order $o\,(G(K,F))$ satisfies $o\,(G(K,F)) \leq [K:F]$.

14. (a) If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

Or

(b) If $(vT, vT) = (v, v)$ for all $v \in V$ then prove that T is unitary.

15. (a) If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then prove that $T = 0$.

Or

(b) Let $T \in A(V)$ be Hermition prove that all its characteristic roots are real.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If V is finite dimensional and if W is a subspace of V, then prove that W is finite dimensional, $\dim W \le \dim V$ and $\dim\left(V/W\right) = \dim V - \dim W$.

17. Derive the Gram - Schmidt orthogonolisation process.

18. Prove that the general polynomial of degree $n \ge 5$ is not solvable by radicals.

19. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

20. For $A, B \in F_n$, Prove that $\det(AB) = (\det A)(\det B)$.

D–7519

Sub. Code
31122

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Riemann-stielties of a bounded real function f on $[a,b]$.

2. Define refinement of a partition.

3. When will you say a function is uniformly bounded?

4. Define rectifiable curve.

5. On what intervals does the series $f(x)=\sum_{n=1}^{\infty}\frac{1}{1+n^2x}$ fail to converge uniformly?

6. Define fourier series.

7. show that the measurable set E is periodic.

8. When will you say a function is Borel measurable?

9. Define Simple function.

10. Let f be a non negative measurable function. Show that $\int f = 0$ implies $f = 0$ almost every where.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b)

11. (a) Prove that $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\in > 0$, there exists a partition P such that $V(P,f,\alpha) - L(P,f,\alpha) < C$.

Or

(b) State and prove schwarz inequality.

12. (a) If K is a compact metric space, if $f_n \in \mathcal{C}(k)$ for $n = 1,2,..,$ and if $\{f_n\}$ converge uniformly on k, then prove that $\{f_n\}$ is equi continuous on k.

Or

(b) Let $a_o, a_1, ..a_n$ be complex numbers, $n \geq 1, a_n \neq 0$ such that $p(z) = \sum_{k=0}^{n} a_k z^k$, prove that $p(z) = 0$ for some complex number z.

13. (a) Prove that, if $x > 0$ and $y > 0$ then $\int_0^1 f^{x-1}(1-t)^{y-1} dt = \frac{\sqrt{x}\sqrt{y}}{\sqrt{x+y}}$

Or

(b) Define Borel set. Prove that every Borel set is measurable.

14. (a) State and prove Lusin's theorem.

Or

(b) If A and B are disjoint measurable sets of finite measure, then prove that $\int_{A \cup B} f = \int_A f + \int_B f$.

15. (a) State and prove Fatou's theorem

Or

(b) State and prove bounded convergence theorem.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b / \gamma'(t) / dt$

17. State and prove the stone-weierstrass theorem

18. Prove

(a) For every $A \in \mathcal{G}$, $\mu * (A) = \mu(A)$

(b) If $E = \bigcup_{n=1}^{\infty} E_n$, then $\mu * (E) \leq \sum_{n=1}^{\infty} \mu * (E_n)$.

19. Prove that the outer measure of an interval is its length.

20. State and prove Lebesgue's dominated convergence theorem.

D–7520

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

Second Semester

Mathematics

TOPOLOGY – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a G_5 – set in a space. Give an example.

2. Define one point compactification.

3. Define countably locally finite.

4. Define locally metrizable space.

5. Give an example of a Cauchy sequence in Q that is not convergent in it.

6. Define the point open topology.

7. Define the evaluation map.

8. Define a Baire space. Give an example.

9. Define a equicontinuous space.

10. What is meant by topological dimension?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a product of a completely regular space is completely regular.

Or

(b) Let $A \subset X$; let $f : A \to Z$ be a continuous map of A into the Hausdorff space Z. Prove that there is atmost one extension of f to a continuous function $g : \overline{A} \to Z$.

12. (a) Let X be a topological space. Prove that the set $\mathcal{B}(X, R)$ of all bounded functions $f : x \to R$ is complete under the sub metric J.

Or

(b) Show that the metric (x, d) is complete if and only if for any nested sequence $A_1 \supset A_2 \supset \ldots$ of non empty closed sets of X such that diameter $A_n \to 0, \bigcap A_n \neq \varphi$ $n \in z_+$.

13. (a) Prove that every para-compact space X is normal.

Or

(b) If X is locally compact, or if X satisfies the first countability axiom, then prove that X is compactly generated.

14. (a) Prove that every metrizable space is para compact.

Or

(b) Show that every open subset of a Baire space is a Baire space.

15. (a) Define $F_\sigma -$ set. Also prove that W is an $F_\sigma -$ set in X if and only if $X - W$ is a $G_\delta -$ set.

Or

(b) Show that any compact subset C of R^2 has topological dimension at most 2.

PART C— (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tychonoff theorem.

17. State and prove the Peano space-filling curve.

18. Prove that a metric space (x, d) is compact if and only if it is complete and totally bounded.

19. State and prove Ascoli's theorem.

20. Let $X = Y \cup Z$, where Y and Z are closed sets in X having finite topological dimension. Prove that $\dim X = \max\{\dim Y, \dim z\}$.

D–7521

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Show that $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ is integrable.

2. Define orthogonal trajectories of a system of curves on a surface.

3. Find the complete integral of $pq = 1$.

4. Eliminate the arbitrary function f from the equation $z = f(x^2 + y^2)$.

5. Find the partial differential equation by the elimination of a and b from $z = (x + a)(y + b)$

6. Write down the fundamental idea of Jacob's method.

7. When we say that the equation $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ is elliptic.

8. Solve $(D^2 - D')z = 0$.

9. Write down the interior Dirichlet boundary value problem for Laplace's equation.

10. Write down the exterior Neumann boundary value problem.

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1, x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

Or

(b) Show that $y\,dx + x\,dy + 2z\,dz = 0$ is integrable and hence solve.

12. (a) Eliminate the arbitrary function f from $z = xy + f(x^2 + y^2)$.

Or

(b) Find the general integral of the partial differential equation $z(xp - yq) = y^2 - x^2$.

13. (a) Find the general solution of the partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ and also the particular integral which passes through the line $x=1, y=0$.

Or

(b) Using Jacobi's method, solve $xp^2+yq^2=z$.

14. (a) Solve $v+s-2t=e^{x+y}$.

Or

(b) Solve $(D'^2+2kD'-C^2D^2)y=0$.

15. (a) Derive D'Alembert's solution of the one-dimensional wave equation.

Or

(b) Is it possible to reduce the Neumann problem to Dirichlet problem? Justify.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Verify that the equation
$z(z+y^2)dx+z(z+x^2)dy-xy(x+y)dz=0$ is integrable and find its primitive.

17. Show that the equations $xp=yq$ and $z(xp+yq)=2xy$ are compatible, and solve them.

18. Reduce the equation $\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$ to canonical form.

19. By separating the variables, solve the one dimensional diffusion equation. $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

20. Heat flows in a semi-infinite rectangular plate, the end $x = 0$ being kept at temperature θ_0 and the long edge $y = 0$ and $y = a$ at zero temperature. Prove that the temperature at a point (x, y) is

$$\frac{4\theta_0}{\pi}\sum_{n=0}^{\infty}\frac{1}{2m+1}\sin\left(\frac{(2m+1)\pi y}{a}\right)e^{-(2m+1)\frac{\pi x}{a}}.$$

D–7522

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define Torsion of curve.

2. What is meant by rectifying plane?

3. Define point of inflexion.

4. What are the intrinsic equations of the curve?

5. Define class of a surface.

6. Define the pitch of the helicoid.

7. Define geodesic parallels.

8. State the necessary and sufficient condition that the curve $v = c$ be a geodesic.

9. State the canonical geodesic equation.

10. Define a developable surface.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If the radius of spherical curvature is constant, then prove that the curve either lies on a sphere or has constant curvature.

Or

(b) Prove that the involute of a circular helix are plane curves.

12. (a) If a curve lies on a sphere, show that σ and ρ are connected by $\frac{\rho}{\sigma} + \frac{d}{ds}(\sigma\rho') = 0$.

Or

(b) Prove that a characteristic property of helix is that the ratio of the curvature to the torsion is constant.

13. (a) Prove that the metric is invariant under a parameter transformation.

Or

(b) Find the co-efficient of the direction which makes an angle $\pi / 2$ with direction where co-efficients are (l, m).

14. (a) Prove that every helix on a cylinder is geodesic.

Or

(b) Prove that on a general surface, a necessary and sufficient condition that the curve $v = c$ be a Geodesic is $EE_2 + FE_1 - 2EF_1 = 0$.

15. (a) Write a brief note on second fundamental form of a surface.

Or

(b) State and prove Meusnier's theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadratic surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$.

17. State and prove uniqueness theorem.

18. (a) Find the differential equations of the orthogonal trajectories of a given family of curves on a given surface.

(b) If θ is the angle of the points uv between the two direction du, dv given by $pdu^2 + 2Qdudv + Rdv^2 = 0$, then prove that $\tan\theta = \frac{2H(Q^2 - PR)^{1/2}}{ER - 2FQ + GP}$.

19. Derive the geodesic differential equations.

20. State and prove the Monge's theorem.

D–7523

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define path, spanning tree.

2. Define cut capacity and event.

3. Define PERT and Float.

4. Define basic solution.

5. Define the optimality condition of the revised simplex method.

6. Define pay-off matrix.

7. Define strong maxima.

8. Define sensitivity coefficients.

9. Define absolute maximum.

10. Define Separable.

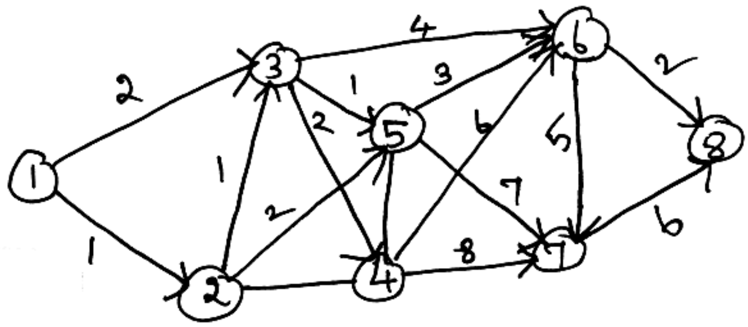
PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11\. (a) Write down the minimal spanning tree algorithm.

Or

(b) Use Dijkstra's algorithm to determine the optimal solution of the following problem.



12\. (a) Determine all basic feasible solution of the following system of equations $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$.

Or

(b) Determine the strategies that define the saddle point and the value of the game.

	B_1	B_2	B_3	B_4
A_1	5	–4	–5	6
A_2	–3	–4	–8	–2
A_3	6	8	–8	–9
A_4	7	3	–9	6

13. (a) Determine the extreme point of the following function

$$f(X) = 2x_1^2 + x_2^2 + x_3^2 + +6(x_1 + x_2 + x_3) + 2x_1x_2x_3.$$

Or

(b) Solve by Jacobian method

Minimize $f(X) = 5x_1^2 + x_2^2 + 2x_1x_2$

Subject to

$g(X) = x_1x_2 - 10 = 0$

$x_1, x_2 \geq 0$

14. (a) Explain the constrained algorithm.

Or

(b) Write down the sufficient condition of the KKT conditions.

15. (a) Construct the project network

Activity	Predecessor (s)	Duration	Activity	Predecessor (s)	Duration
A	-	3	I	H	3
B	A	14	J	H	2
C	A	1	K	I, J	2
D	C	3	L	K	2
E	C	1	M	L	4
F	C	2	N	L	1
G	D, E, F	1	O	B, M, N	3
H	G	1			

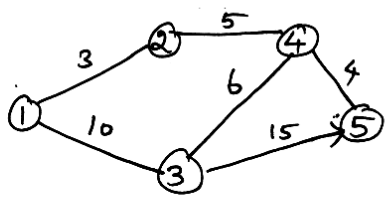
Or

(b) Explain the critical path method.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. For the following network find the shortest route between every two nodes.



17. Solve the following LPP by revised simplex method
Minimize $Z = 2x_1 + x_2$
Subject to
$3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \leq 3$
$x_1, x_2 \geq 0$

18. Solve the following game by Linear programming technique

		Player Q		
		Q_1	Q_2	Q_3
Player P	P_1	9	1	4
	P_2	0	6	3
	P_5	5	2	8

19. Solve $f(x) = (3x-2)^2\ (2x-3)^2$ by Newton Raphson method.

20. Solve the following LPP by Lagrangean method
Minimize $f(X) = x_1^2 + x_2^2 + x_3^2$
Subject to
$g_1(X) = x_1 + x_2 + 3x_3 - 2 = 0$
$g_2(X) = 5x_1 + 2x_2 + x_3 - 5 = 0$

D–7524

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. If $(a, b) = 1$ and if $c \mid a$ and $d \mid b$, then show that $(c, d) = 1$.

2. Find the g.c.d. of 826, 1890.

3. Define Mangoldt function $\wedge$ and write down the values of $\wedge(n)$, for $1 \leq n \leq 5$.

4. Define Multiplicative functions.

5. If f and g are arithmetical functions then prove that $(f * g)' = (f' * g) + (f * g')$.

6. Define the big *oh* notation and asymptotic equality of functions.

7. State the Little Fermat's theorem.

8. Solve the congruence $25x \equiv 15 \pmod{120}$.

9. Define Legendre's symbol. Give an example.

10. Determine whether –104 is a quadratic residue or non-residue of the prime.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

Or

(b) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

12. (a) For any two arithmetical functions f and g, let $h = f * g$. Prove that, for every prime p, $h_p(x) = f_p(x) g_p(x)$.

Or

(b) Prove that for every $n \geq 1$, $\sum_{d|n} \lambda(d) = \begin{cases} 1 & \text{if } n \text{ is a square} \\ 0 & \text{Otherwise} \end{cases}$ Also prove that $\lambda^{-1}(n) = |\mu(n)|$ for all n.

13. (a) If f is multiplicative then prove that $\sum_{d|n} \mu(d) f(d) = \prod_{p|n} (1 - f(p))$.

Or

(b) State and prove Generalized inversion formula.

14. (a) If a and b are positive real numbers such that $ab = x$ then prove that

$$\sum_{\substack{q,d \\ qd \le x}} f(d)\, g(d) = \sum_{n \le a} f(n)\ G\left(\frac{x}{n}\right) + \sum_{n \le b} g(n)\, F\left(\frac{x}{n}\right) - F(a)G(b).$$

Or

(b) State and prove Euler-Fermat theorem.

15. (a) State and prove Chinese remainder theorem.

Or

(b) If P and Q are positive odd integers with $(P, Q) = 1$ then prove that $\left(\frac{P}{Q}\right)\left(\frac{Q}{P}\right) = \frac{(-1)^{(p-1)(Q-1)}}{4}$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. (a) State and prove Euclidean algorithm.

(b) Prove that if $n \ge 1$, $\sum_{d|n} \varphi(d) = n$.

17. State and prove product form of the Mobius inversion formula.

18. Let f be multiplicative. Prove that f is completely multiplicative if and only if, $f^{-1}(n) = \mu(n) f(n)$, for all $n \ge 1$.

19. (a) Prove that congruence is an equivalence relation.

(b) Assume $(a, m) = 1$. Then prove that the linear congruence $ax \equiv b \pmod{m}$ has exactly one solution.

20. State and prove Gauss Lemma.

D–7525

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define null persistent state.

2. Define Markov chain.

3. Define residual time.

4. Define diffusion processes.

5. Write the backward diffusion equation.

6. Define Ornstein-Vhlenbeck process.

7. Define service time.

8. State the total number of progeny.

9. Define Idle period.

10. Write the Erlang's loss formula.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11\. (a) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2, with transition matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution $\Pr(X_0 = i) = \frac{1}{3}, i = 0, 1, 2$

Find

(i) $\Pr(X_2 = 2, X_1 = 1 / X_0 = 2)$

(ii) $\Pr(X_2 = 2, X_1 = 1, X_0 = 2)$

Or

(b) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that $\Pr\{N(s) = k / N(t) = n\} = \binom{n}{k}\left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$

12\. (a) If $X(t)$, with $X(0)$ and $\mu = 0$, is a Wiener process, show that $Y(t) = \sigma X\left(\frac{t}{\sigma^2}\right)$ is a Wiener process. Find its covariance function.

Or

(b) Let $X(t)$ be the displacement process corresponding to the velocity $\mu(t)$ show that in equilibrium position $E[X(t) - X(0)] = 0$ and $Var[X(t) - X(0)] = \sigma^2(\beta t + e^{-\beta t} - 1) / \beta^3$.

13. (a) For a G.W. process with $m=1$, $\sigma^2 < \infty$, prove that $\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{1-P_n(s)} - \frac{1}{1-s}\right\} \to \frac{\sigma^2}{2}$ uniformly in $0 \le s < 1$.

Or

(b) Find the variance of X_n in a G.W process from the relation $P_n = P(P_{n-1}(s))$

14. (a) Derive Little's formula.

Or

(b) For $r, n = 0, 1, 2, \ldots\ldots;$ prove that $E\{E_{n+r} / X_n\} = X_n m^r$.

15. (a) Show that the expected number of busy servers in an $M/M/C$ queue in steady state is $CP = \lambda/\mu$.

Or

(b) Derive Pollaczek-Khinchine formula.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Prove that the p.g.f. of a non-homogeneous process $\{N(t), t \ge 0\}$ is $Q(s,t) = \exp\{m(t)(s-1)\}$, where $m(t) = \int_0^1 \lambda(x)\,dx$ is the expectation of $N(t)$.

17. If the intervals between successive occurrences of an event E are independently distributed with a common exponential distribution with mean $\frac{1}{\lambda}$, then prove that the events E form a Poisson process with mean λt.

18. Discuss briefly and derive Ornstein-Vhlenbeck process.

19. State and prove Yaglom's theorem.

20. A mechanic looks after 8 automatic machines, a machine breaks down, independently of others, in accordance with a Poisson process, the average length of time for which a machine remains in working order being 12 hours. The duration of time required for repair of a machine has an exponential distribution with mean 1 hour. Find

    (a) The probability that 3 or more machines will remain out of order at the same time.

    (b) For what fraction of time, on the average, the mechanic will be idle and

    (c) The average duration of time for which a machine is not in working order.

D–7526

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define degree of a vertex in a graph.

2. Define walk and path of a graph.

3. Define edge connectivity. Give an example.

4. Define the edge chromatic number.

5. Prove that in a tree any two vertices are connected by a unique path.

6. Find the number of different perfect matchings in k_n.

7. If G is a simple planar graph with $\gamma \geq 3$, then prove that $\varepsilon \leq 3\gamma - 6$.

8. Using Euler's formula, prove that, if G is a simple planar graph, then $\delta \leq 5$.

9. Define isomorphic directed graphs.

10. Define directed path and directed cycles.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In any graph G, prove that the number of vertices of odd degree is even.

Or

(b) Show that $k_p - v = k_{p-1}$ for any vertex v of k_p.

12. (a) If $\delta \geq k$ then prove that G has a path of length K.

Or

(b) Let G be a K-regular bipartite.

13. (a) Prove that $\gamma(m,n) = \gamma(n,m)$.

Or

(b) If G is bipartite then prove that $\chi' = \Delta$.

14. (a) Show that K_5 is not planar.

Or

(b) When will you say a graph is self dual? Also prove that if G is self dual, then $\varepsilon = 2V = 2$.

15. (a) Prove that a graph G is strongly orientable if and only if G has no cut edge.

Or

(b) Prove that every tournament has a spanning path.

PART C— (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that $\tau(k_n) = n^{n-2}$.

17. State and prove Vizing's theorem.

18. State and prove Euler's formula for a connected plane graph.

19. State and prove Brook's theorem.

20. Prove that a weak digraph D is Eulerian if and only if every point of D has equal indegree and outdegree.

D–7527

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define normed space. Give an example.

2. Define orthogonal transformation.

3. Define the dimension of a Hilbert space.

4. Let X be a linear space over c and u a real-linear functional on X. Define $f(x) = ux - iu(ix),\ x \in X$. Prove that f is a complex linear functional on X.

5. Write a note on Banach limit.

6. Define annihilator.

7. Define unitary and normal operators.

8. Show that l^2 is reflexive.

9. State the open mapping theorem,

10. Let $x \in X$, X a linear space. Prove that $< x, y >= 0$ for all $y \in X$ if and only if $x = 0$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (X,d) and (Y,d') are metric spaces where $f:X \to Y$, then prove that f is continuous at x if and only if for every sequence $\{x_n\}$ converging to x, $f(x_n) \to f(x)$.

Or

(b) Prove that norm is a continuous function.

12. (a) Let X be a normed linear space and let k be a convex subset of X. Prove that $\overline{K}$ is convex.

Or

(b) Let X and Y be normed linear spaces. Let $T:X \to Y$ be a linear transformation. Prove that T is continuous if and only if T is continuous at the origin.

13. (a) Prove that a normed space with Schauder basis is separable.

Or

(b) Prove that every inner product space is a normed linear space.

14. (a) Prove that any two orthonormal bases in a Hilbert space have the same cardinality.

Or

(b) Let $\widetilde{X}$ be the dual space of the normed linear space X. Prove that if $\widetilde{X}$ is separable, so is X.

15. (a) Suppose $A : X \to Y$. If A is completely continuous then prove that the range of A, $R(A)$ is separable.

Or

(b) Show that a completely continuous transformation maps a weakly convergent sequence into a strongly convergent one.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Prove that a subset A of a metric space (X, d) is sequentially compact if and only if it is compact.

17. Let X be an inner product space, A an orthonormal set of vectors in X, and y an arbitrary vector in X. Then prove that

(a) for all $x_1, x_2 ...., x_n \in A$, $\sum_{i=1}^{n} \left| (y, x_i) \right|^2 \leq \| y \|^2$

(b) the set $E = \{x \in A / (y, x) \neq 0\}$ is countable

(c) if $z \in X$, then $\sum_{x \in A} \left| (y, x) \overline{(z, x)} \right| \leq \| y \| \cdot \| z \|$.

18. State and prove Riesz representation theorem for a linear functionals on a Hilbert space.

19. State and prove uniform boundedness theorem.

20. State and prove closed graph theorem.

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D–7528

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2022.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a multiple root of multiplicity m.

2. What is meant by strum sequence?

3. Define eigen vector.

4. What is meant by truncation error?

5. State the Hermite interpolating polynomial.

6. Write the Lagrange bivariate interpolating polynomial.

7. Define Gauss elimination method.

8. What is the order of the error in Simpson's formula?

9. What are the two types of errors in numerical differentiation?

10. Write the advantages of R-K model.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve $x^3 - 5x + 1 = 0$ by the Newton-Raphson method.

Or

(b) Use synthetic division and perform two iterations of the Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$.

12. (a) Solve the following equations using the Gauss elimination method.

$10x - y + 2z = 4$

$x + 10y - z = 3$

$2x + 3y + 20z = 7.$

Or

(b) Find the largest eigen value in modulus and the corresponding eigen vectors of the matrix.

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

13. (a) Given that

x :	0	1	2	3
$f(x)$:	1	2	33	244

Fit quadratic splines with $M(0) = f''(0)$. Hence, find an estimate of $f(2.5)$.

Or

(b) Obtain the least squares straight line fit to the following data :

x :	0.2	0.4	0.6	0.8	1
$f(x)$:	0.447	0.632	0.775	0.894	1

14. (a) A differentiation rule of the form

$hf'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$ where $x_j = x_0 + jh,\ j = 0,1,2,3,4$ is given. Determine the values of $\alpha_0,\ \alpha_1,\ \alpha_2,\ \alpha_3$ so that the rule is exact for a polynomial of degree 4 and find the error term.

Or

(b) Determine $\alpha, \beta, \gamma, \delta$ so that the relation $y'((a+b)/2) = \alpha y(a) + \beta y(b) + \gamma y''(a) + \delta y''(b)$ is exact for polynomials of as high degree as possible. Give an asymptotically valid expression for the truncation error as $|b-a| \to 0$.

15. (a) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using trapezoidal rule and Simpson's rule.

Or

(b) Evaluate $\int_0^\infty \frac{e^{-x}}{1+x^2}\,dx$, using the Gauss-Laguerre two point formula.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the interval in which the smallest positive root of the equation $x^3 - x - 4 = 0$ lies. Determine the roots current to two decimal places using the bisection method.

17. Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $p_3(x) = x^3 + x^2 - x + 2 = 0$. Use the initial approximations $p_0 = -0.9, q_0 = 0.9$.

18. Construct the Hermite interpolation polynomial that fits the data

x	$f(x)$	$f'(x)$
0	4	–5
1	–6	–14
2	–22	–17

Interpolate $f(x)$ at $x = 0.5$ and $x = 1.5$.

19. Solve the initial value problem $u' = -2tu^2, u(0) = 1$ using the midpoint method, with $h = 0.2$, over the interval $[0, 1]$. Use Taylor series method of second order to compute $u\ (0.2)$.

20. Using fourth order Runge-Kutta method find $u(0.2)$ from $u' = u - t,\ u(0) = 2$ taking $h = 0.1$.

D–7529

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Find $P(A/B)$ and $P(B/A)$ if $P(A) = 0.4, P(B) = 0.5$ and $P(A \cap B) = 0.3$.

2. Define the conditional expectation.

3. Define moment generating function.

4. Define an exponential distribution.

5. If the moment generating function of a random variable X is $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^5$, find $P(x = 2 \; or \; 3)$.

6. Write down the mean and variance of the beta distribution.

7. Let X be a $N(2.25)$ Find $P_r(0 < x < 10)$.

8. Let the independent random variables x_1 and x_2 have the same p.d.f. $f(x) = \begin{cases} \frac{x}{6}, & x = 1, 2, 3 \\ 0 & elsewhere \end{cases}$, find $P_r(x_1 = 2, x_1 = 3)$.

9. Let y be b $(72, \frac{1}{3})$. Find $P_r(22 \le y \le 28)$.

10. Let z_n be $x^2(n)$. Find the mean and variance of z_n.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Baye's theorem.

Or

(b) A bowl contains seven blue chips and three red chips. Two chips are to be drawn successively at random and without replacement. Compute the probability that the first draw results in a red chip (A) and the second draw results in a blue chip (B).

12. (a) Prove that

(i) $E[E(x_2 / x_1)] = E(x_2)$

(ii) $Var\,[E(x_2 / x_1)] \le \text{var}\,(x_2)$

Or

(b) Let the joint p.d.f. of x_1 and x_2 be

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, 0 < x_1 < 1; & 0 < x_2 < 1 \\ 0, & elsewhere \end{cases}$$

Show that the random variables x_1 and x_2 are dependent.

13. (a) Derive the mean and variance of a gamma distribution.

Or

(b) Let x equal a tarsus length for a male grackle. Assume that the distribution of x is $N(\mu, 4.84)$. Find the sample size n that is needed so that we are 95% confident that the maximum error of the estimate of μ is 0.4.

14. (a) Let $\overline{X}$ be the mean of a random sample of size 12 from the uniform distribution on the interval (0,1). Find $P(1/2 \leq \overline{X} < 2/3)$.

Or

(b) Let x have the uniform distribution over the interval $(-\pi/2, \pi/2)$. Show that $y = \tan x$ has a cauchy distribution.

15. (a) Let the p.d.f. Y_n be $f_n(y) = \begin{cases} 1, & y = n \\ 0, & elsewhere \end{cases}$ show that Y_n does not have a limiting distribution.

Or

(b) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. (a) State and prove Chebyshev's inequality.

(b) Prove that $E[(x - \mu_1)(y - \mu_2)] = E(xy) - \mu_1 \mu_2$

17. Let x_1 and x_2 have the joint p.d.f.

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x < 1, 0 < x_2 < 1 \\ 0, & elsewhere \end{cases}$$

Find

(a) $P(x_1 \leq 1/2)$

(b) $P_r(x_1 + x_2 \leq 1)$

18. Let S^2 be the variance of random sample of size 6 from the normal distribution $N(\mu, 12)$. Find $P(2.30 < S^2 < 22.2)$

19. Find the p.d.f. of the beta distribution in which α and β are parameters.

20. State and prove central limit theorem.

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D–2190

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

ALGEBRA – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define relatively prime integers. Give an example.

2. Define a subgroup of a group G. Give an example.

3. Define the centre of a group G.

4. Define an internal direct product of groups.

5. Show that the group of order 21 is not simple.

6. Define a ring homomorphism.

7. If D is an integral domain with finite characteristic, prove that the characteristic of D is a prime.

8. Define a left-ideal of R.

9. If $p(x)=1+x-x^2$ and $q(x)=2+x^2+x^3$, then find $p(x)q(x)$.

10. Prove that an Euclidean ring posses an unit element.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If a is relatively prime to b but $a \mid bc$, then prove that $a \mid c$.

Or

(b) State and prove the Cauchy's theorem for abelian group.

12. (a) Show that the subgroup N of G is a normal subgroup of G if and only if every left-coset N in G. is a right coset N in G.

Or

(b) State and prove Lagrange's theorem.

13. (a) Prove that a ring homomorphism $\phi : R \to R'$ is one-to-one if and only if the Kernal of ϕ is zero submodule.

Or

(b) Prove that a finite integral domain is a field.

14. (a) If $[a, b] = [a', b']$ and $[c, d] = [c', d']$ then prove that $[a, b][c, d] = [a', b'][c' d']$.

Or

(b) Let R, R' be rings and ϕ a homomorphism of R onto R' with Kernal V. Prove that R' is isomorphic of R/V.

15. (a) State and prove the Einstein criterion theorem.

Or

(b) If R is an Euclidean domain, prove that any two elements a and b in R have greatest common divisor.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Cayley's theorem.

17. If p is a prime number and $\frac{p^{\alpha}}{D(G)}$, then prove that G has a subgroup of order p^{α}.

18. Define an integral domain and a Euclidean ring. Prove that any field is an integral domain.

19. If R is a unique factorization domain, then prove that $R[x]$ is also an unique factorization domain.

20. (a) State and prove Gauss lemma.

(b) State and prove the division algorithm.

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D–2191

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define bounded set. Give an example.

2. Prove that $z\,\bar{z}$ is real and positive, except when $z = 0$, where z is a complex number.

3. Define convex set.

4. Prove that in a metric space convergence sequence is bounded.

5. Define Cauchy sequence.

6. Prove that $\lim_{n \to \infty} \sqrt[n]{n} = 1$.

7. Define discontinuity of the first kind at x.

8. Define closed balls in R^n.

9. State the mean value theorem.

10. Define continuously differentiable function.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $x \in R, y \in R$ and $x < y$, then prove that there exists a $P \in Q$ such that $x < p < y$.

Or

(b) Let z and w be complex numbers. Prove that $|z + w| \leq |z| + |w|$.

12. (a) Prove that every infinite subset of a countable set is countable.

Or

(b) Prove that closed subsets of compact sets are compact.

13. (a) If X is a compact metric space and if $\{p_n\}$ is a cauchy sequence in X, then prove that $\{p_n\}$ converges to some point of X.

Or

(b) Prove that, a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

14. (a) Let f be a continuous real valued function on a metric space X. Let $Z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Prove that $z(f)$ is closed.

Or

(b) Let f be monotonic on (a, b). Prove that the set of points of (a, b) at which f is discontinuous is at most countable.

15. (a) State and prove intermediate value theorem.

Or

(b) State and prove Rolle's theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that every k-cell is compact.

17. Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

18. Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

19. State an prove Heine-Borel covering theorem.

20. State and prove Taylor's theorem.

D–2192

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Find all real valued solutions of the equation $y'' - y = 0$.

2. Find two linearly independent solutions of $y'' + \frac{1}{4x^2} y = 0$ $(x > 0)$.

3. State the existence theorem for initial value problem.

4. Compute the Wronskian of $\phi_1(x) = x^2$, $\phi_2(x) = 5x^2$.

5. Define indicial polynomial.

6. Find the singular points of the equation $x^2 y'' + (x + x^2) y' - y = 0$.

7. Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y' = y^2$, $y(0) = 0$.

8. Find the integrating factor of the equation

$\cos x \cos y\, dx - 2 \sin x \sin y\, dy = 0$.

9. State the local existence theorem for the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on z.

10. When you say that a solution exists non-locally?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $L(y) = 0$ be an n^{th} order differential equation on an interval I. Show that there exist linearly independent solution of $L(y) = 0$ on I.

Or

(b) Find all the solutions of the equation :

$y'' - 4y' + 5y = 3e^{-x} + 2x^2$.

12. (a) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

Or

(b) Show that the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n\left[(n!)^2\right]}$.

13. (a) Prove that, between any two positive zeros of J_0 there is a zero of J_1.

Or

(b) Show that -1 and $+1$ are the regular singular points of the Legendre equation $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$.

14. (a) Find the integrating factor of $(2xy^3+2)dx+3xy^2dy=0$ and solve it.

Or

(b) Prove that a function ϕ is a solution of the initial value problem $y'=f(x, y)$, $y(x_0)=y_0$ on an interval I if and only if it is a solution of the integral equation $y=y_0+\int_{x_0}^{x} f(t, y)dt$ on z.

15. (a) Let f be a real valued continuous function on the strip $S:|x|\leq\infty, |y|<\infty$, $\alpha<0$, and suppose f satisfies a lipschitz condition on S. Show that the solution of the initial value problem $y'+\lambda^2 y=f(x, y), y(0)=0$, $y'(0)=1, \lambda>0$ is unique.

Or

(b) Consider $y_1' = 3y_1 + xy_3,\ y_2' = y_2 + x^3 y_3,\ y_3' = 2xy_1 - y_2 + e^x y_3$. Show that every initial value problem for this system has a unique solution which exists for all real x.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the two linearly independent power series solution of the equation $y''-xy'+y=0$.

17. With usual notations, prove that $J_0(x)=\sum_{m=0}^{\infty}\frac{(-1)^m}{(m!)^2}(x/2)^{2m}$.

18. If ϕ_1 is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I, and $\phi_1(x) \neq 0$ on I, prove that a second solution $\phi_2(x) = \phi_2(x)\int_{x_0}^{x} \frac{1}{[\phi(s)]^2} \exp\left[-\int_{x_0}^{x} a_1(t)dt\right] ds$. Also prove that the functions ϕ_1, ϕ_2 form a basis for the solutions of $L(y) = 0$ on I.

19. Derive the indicial polynomial for the Euler equation.

20. Let f be a real valued continuous function on the strip $S: |x - x_0| \leq \alpha,\ |y| < \infty \ (a > 0)$ and suppose that f satisfies on S a Lipschitz condition with constant $k > 0$. Prove that the successive approximation $\{\phi_k\}$ for the problem $y' = f(x, y),\ y(x_0) = y_0$ exist on the entire interval $|x - x_0| \leq a$ and converge these to a solution ϕ of $y' = f(x, y),\ y(x_0) = y_0$.

D–2193

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define onto function. Give an example.
2. Define uncountable set. Give an example.
3. State the well ordering theorem.
4. Define the discrete topology. Give an example.
5. What is meant by the product topology?
6. Define connected and path connected sets.
7. Define compact space.
8. State the tube lemma.
9. Define first countability axiom.
10. Define normal space.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a countable union of countable sets is countable.

Or

(b) Prove that every non-empty finite ordered set has the order type of a section $\{1, 2, ..., n\}$ of $\mathbf{Z}_+$, so it is necessarily welt ordered.

12. (a) Let X be a topological space; let A be a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

Or

(b) Show that lower limit topology τ' on R is strictly finer than the standard topology τ.

13. (a) Let Y be a subspace of X. Prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

Or

(b) Prove that every finite point set in a Hausdorff space X is closed.

14. (a) Prove that the union of a collection of connected sets that have a point in common is connected.

Or

(b) State and prove maximum and minimum value theorem.

15. (a) Show that if X is Lindelöf and Y is compact, then $X \times Y$ is Lindelöf.

Or

(b) Prove that every compact Hausdorff space is normal.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If A is finite, then prove that there is no bijection of A with a proper subset of itself.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric δ are the same as the product topology on $\mathbb{R}^n$.

18. State and prove the sequence lemma.

19. If L is a linear continuum in the order topology, then prove that L is connected and so is every interval and ray in L.

20. State and prove the Urysohn Lemma.

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D–2194

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. In a vector space, show that $\infty(v-w)=\infty v-\infty w$.

2. Define subspace of a vector space. Give an example.

3. Define inner product space.

4. Define an orthonormal set.

5. Define an algebraic number.

6. For any $f(x), g(x) \in F[x]$ and any $\infty \in F$, prove that $(f(x)+g(x))' = f'(x)+g'(x)$.

7. Express the polynomial $x_1^3+x_2^3+x_3^3$ in the elementary symmetric functions in x_1, x_2, x_3.

8. Define the range of T.

9. Define self adjoint and skew-Hermitian.

10. Define characteristic root of T.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the intersection of two subspaces of V is a subspace of V.

Or

(b) Prove that $\|\alpha u\| = |\alpha| \|u\|$.

12. (a) State and prove Schwarz inequality.

Or

(b) If $\{V_i\}$ is an orthonormal set, then prove that the vectors in $\{V_i\}$ are linearly independent.

13. (a) State and prove remainder theorem.

Or

(b) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

14. (a) Prove that K is a normal extension of F if and only if K is the spilitting field of some polynomial over F.

Or

(b) Prove that $G(K, F)$ is a subgroup of the group of all automorphism of K.

15. (a) If $< vT, vT > = < v,v >$ for all $v \in V$, then prove that T is unitary.

Or

(b) If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If $v_1, v_2, ..., v_n$ is a basis of V over F and if $w_1, w_2, ..., w_m$ in V are linearly independent over F, then prove that $m \leq n$.

17. Let V be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

18. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F, moreover, prove that $[L : F] = [L : K][K : F]$.

19. State and prove the fundamental theorem of Galoi's theory.

20. For every prime number p and every positive integer m there is a unique field having p^m elements.

D–2195

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Riemann – Stieltjes integral.

2. Define rectifiable curve.

3. Give an example of a sequence which is not equicontinuous.

4. Show that e^x is continuous and differentiable for all x.

5. Define equicontinuous families of functions with an example.

6. Prove that the function E is periodic, with period 2π i.

7. If $m * E = 0$, then prove that E is measurable.

8. Define outer measure.

9. Define Lebesgue integral.

10. With usual notations, prove that $|f| = f^+ + f^-$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If p^* is a refinement of p, then prove that $L(p, f, \alpha) \leq L(p^*, f, d)$.

Or

(b) State and prove integration by parts theorem.

12. (a) State and prove the Fundamentals theorem of calculus.

Or

(b) Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$ converges uniformly in every bounded interval, but does not converges absolutely for any value of x.

13. (a) Suppose Σc_n converges. Put

$f(x) = \sum_{n=0}^{\infty} c_n x^n$, $(-1 < x < 1)$. Prove that

$\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

Or

(b) Define the gamma function $\Gamma(x)$ and prove that $\Gamma(x+1) = x\Gamma(x)$, $\Gamma(1) = 1$ and $\log \Gamma(x)$ is convex.

14. (a) Show that the interval (a, ∞) is measurable.

Or

(b) Let $\{A_n\}$ be a countable collection of sets of real numbers. Prove that $m^*\left(\bigcup A_n\right) \leq \sum m^* A_n$.

15. (a) State and prove Fatou's lemma.

Or

(b) Suppose $f = f_1 + f_2$, where $f_1 \in L(\mu)$ on $E(i = 1, 2)$. Prove that $f \in L(\mu)$ on E, and $\int_E f\, d\mu = \int_E f_1 d\mu + \int_E f_2 d\mu$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

17. State and prove the Stone-Weierstrass theorem.

18. State and prove Parseval's theorem.

19. State and prove Lusin's theorem.

20. State and prove Lebesgue's monotone convergence theorem.

D–2196

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

TOPOLOGY – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define the one point compactification. Give an example.
2. What is meant by completely regular space?
3. State countable intersection property.
4. Define an open refinement. Give an example.
5. Define the stone – cech compactification.
6. When will you say cauchy sequence is complete?
7. Define an equicontinuous function.
8. State totally bounded in a metric space.
9. Define Baire space with an example.
10. Define an M-cube.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define locally compact with an example. Show that the rationals Q are not locally compact?

Or

(b) Prove that a subspace of a completely regular space is completely regular.

12. (a) Under what conditions does a metrizable space have a metrizable compactification.

Or

(b) Prove that every paracompact Hausdorff space X is normal.

13. (a) Define F_σ – set. Also prove that W is an F_σ – set in X if and only if $X - W$ is a G_δ set.

Or

(b) Prove that the Euclidean space R^K is complete in either of its usual metrics, the Euclidean metric d or the square metric δ.

14. (a) Show that in the compact open topology, $\zeta(X, Y)$ is Hausdorff if Y is Hausdorff and regular if Y is regular.

Or

(b) Prove that any open subset Y of a Baire space X is itself a Baire space.

15. (a) If Y is a closed subset of X, and if X has finite dimension then so does Y; and $\dim Y \leq \dim X$: Prove.

Or

(b) Prove that every compact subset of $\mathbb{R}^N$ has topological dimension at most N.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that an arbitrary product of compact space is compact in the product topology.

17. State and prove Tietze extension theorem.

18. State and prove the Smirnov metrization theorem (necessary part).

19. State and prove Baire category theorem.

20. Let $h : [0, 1] \rightarrow R$ be a continuous function. Given $\in > 0$, prove that there is a function $g : [0, 1] \rightarrow R$ with $|h(x) - g(x)| < \in$ for all x, such that g is continuous and nowhere – differentiable.

D–2197

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define exact differential equation.

2. Show that the equation $(x^2 z - y^3)dx + 3xy^2 dy + x^3 dz = 0$ is integrable.

3. Define oblique trajectory.

4. Solve : $2p + 3q = 1$.

5. What is the general solution of

   $p_1 p_1 + p_2 p_2 + ... + p_n p_n = R$?

6. Write the subsidiary equations for the equation $\frac{y^2 z}{x} + z\,xy = y^2$.

7. Show that the differential equations $\frac{\partial z}{\partial x} = 5x - 7y$ and $\frac{\partial z}{\partial y} = 6x + 8y$ are not compatible.

8. Solve : $(D^2 - 5DD' + 4D'^2)z = 0$.

9. Solve $\frac{\partial^2 z}{\partial x^2} = 6x$.

10. Write down the interior Dirichlet boundary value problem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve : $\frac{dx}{xz(z^2 + xy)} = \frac{dy}{-yz(z^2 + xy)} = \frac{dz}{x^4}$.

Or

(b) Solve : $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{bx - ay}$.

12. (a) Show that $(2x + y^2 + 2xz)dx + 2xy\,dy + x^2 dz = 0$ is integrable.

Or

(b) Form a partial differential equation by eliminating the arbitrary function f from the equation $x + y + z = f(x^2 + y^2 + z^2)$.

13. (a) Find the integral surface of the linear partial differential equation

$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0,\ z=1$.

Or

(b) Find the family of surfaces orthogonal to family of surfaces given by the differential equation $(y+z)p+(z+x)q = x+y$.

14. (a) Solve : $p_1 + p_2 + p_3 = 4z$.

Or

(b) Find the complete integral of $py + qx + pq = 0$.

15. (a) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

Or

(b) Derive D'Alembert's solution of one-dimensional wave equation.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the orthogonal trajectories of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is a parameter.

17. Solve : $\frac{dx}{x+y-xy^2} = \frac{dy}{x^2y-x-y} = \frac{dz}{z(y^2-x^2)}$.

18. Obtain the complete integral of $p^2 + q^2 - 2px - 2qy + 1 = 0$.

19. Solve : $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$.

20. A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = y_0 \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement at time t.

D–2198

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Osculating plane.

2. What is meant by Torsion?

3. Define evolute.

4. Define are length.

5. Define archor ring.

6. What is meant by the binormal line?

7. Define a hyperbolic point.

8. Write short notes on the geodesic curvature.

9. Define principal curvatures.

10. Define the osculating developable of the curve.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Serret-Frenet formula.

Or

(b) With the usual notations prove that $[\vec{r}', \vec{r}'', \vec{r}'''] = k^2 r$.

12. (a) If a curve lies on a sphere, show that δ and σ are related by $\frac{d}{ds}(\sigma\delta') + \frac{\delta}{\sigma} = 0$.

Or

(b) Show that the involutes of a circular helix are plane curves.

13. (a) Calculate the fundamental coefficients E, F, G and H for the paraboloid $\vec{r} = (u, v, u^2 - v^2)$.

Or

(b) Discuss the geodesic parallels.

14. (a) Find the Gaussian curvature at (u, v) of the achor ring.

Or

(b) Enumerate the geodesic polar form. Also find the circumference of a geodesic circle of small radius r.

15. (a) State and prove Euler's theorem.

Or

(b) Show that the characteristics point of the plane u is determined by the equations $\vec{r} \cdot \vec{a} = p$, $\vec{r} \cdot \vec{a} = \dot{p}, \vec{r} \cdot \ddot{\vec{a}} = \ddot{p}$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Show that the intrinsic equations of the curve given by $x = a e^{u} \cos u$, $y = a e^{u} \sin u$, $z = b e^{u}$ are $k = \dfrac{\sqrt{2}\, a}{(2a^2 + b^2)^{1/2}} \cdot \dfrac{1}{s}$, $\tau = \dfrac{b}{(2a^2 + b^2)^{1/2}} \cdot \dfrac{1}{s}$.

17. Derive the differential equations for a geodesic using the normal property.

18. Find the surface of revolution which is isometric with a region of the right helicoids.

19. State and prove the Gauss-Bonnet theorem.

20. Show that a necessary sufficient condition for a surface to be developable is that its Gaussian curvature shall be zero.

D–2199

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is meant by connected network?

2. Define the total float and the free float.

3. Write two features of O.R.

4. Define critical activity.

5. Define critical path.

6. What is minimax strategy?

7. What is interval of uncertainty?

8. Explain dichotomous search method.

9. Define general constrained non-linear programming problem.

10. What is meant by steepest method?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain three jug puzzle with an illustration.

Or

(b) Construct the network diagram comprising activities B, C,…,Q and N such that following constraints are satisfied.

B < E, F; C < G, L; E, G < H; L < H < I; L < M; H < N, H < J; I, J < P; P < Q. The notation $x < y$. Means that the activity x must be finished before y can begin.

12. (a) Solve :

Minimize : $z = x_1 + x_2 + 2x_3$

Subject to the constraints :

$$\begin{aligned} x_1 + x_2 + x_3 &\le 9 \\ 2x_1 - 3x_2 + 3x_3 &= 1 \\ -3x_1 + 6x_2 - 4x_3 &= 3; \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

Or

(b) Solve :

Maximize $z = 5x_1 + 2x_2$

Subject to the constraints :

$$6x_1 + x_2 \geq 6$$
$$4x_1 + 3x_2 \geq 12$$
$$x_1 + 2x_2 \geq 4$$
$$x_1, x_2 \geq 0.$$

13. (a) Solve the following game and determine the value of the game :

Player B

$$\text{Player A} \begin{pmatrix} 2 & 5 \\ 7 & 3 \end{pmatrix}$$

Or

(b) Is the following two-person zero-sum game stable? (The payoff is for player A). Solve the game problem :

Player B

$$\text{Player A} \begin{pmatrix} 8 & 6 & 2 & 8 \\ 8 & 9 & 4 & 5 \\ 7 & 5 & 3 & 5 \end{pmatrix}$$

14. (a) Find the minimax and maximin for the following data :

$$\begin{pmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{pmatrix}$$

Or

(b) Solve the LPP by Lagrangion method :

Maximize : $f(x) = 5x_1 + 3x_2$

Subject to :

$g_1(x) = x_1 + 2x_2 + x_3 - 6 = 0$
$g_2(x) = 3x_1 + x_2 + x_4 - 9 = 0$
$x_1, x_2, x_3, x_4 \geq 0.$

15. (a) Prove that a necessary condition for X_0 to be an extreme point of $f(X)$ is that $\nabla f(x_0) = 0$.

Or

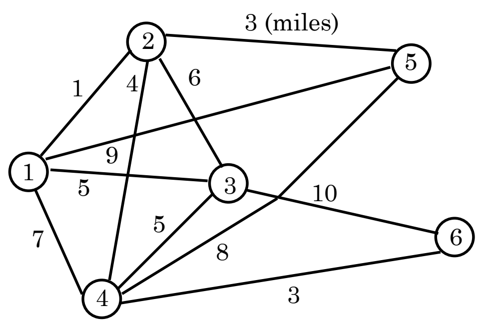
(b) Explain a quadratic programming model problem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the minimal spanning tree of the following network under the independent conditions :

Nodes 2 and 6 are linked by a 4-mile cable.



D–2199

17. The footings of a building can be completed in four consecutive sections :

The activities for each section include digging, placing steel, and pouring concrete. The digging of one section cannot start until the proceeding one is completed. The same restriction applies to pouring concrete. Develop a network for the project.

18. Solve the problem by the revised simplex method :

Minimize : $z = 2x_1 + x_2$

Subject to :

$3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \leq 3$
$x_1, x_2 \geq 0.$

19. Apply the Newton-Raphason method to solve :

$f(x) = 2x_1^2 + x_2^2 + x_3^2 + 6(x_1 + x_2 + x_3) + 2x_1 x_2 x_3$.

Examine the functions for extreme points.

20. Write the Kuhn-Tucker necessary conditions for the problem :

Maximize $f(x) = x_1^3 - x_2^2 + x_1 x_3^2$

Subject to :

$x_1 + x_2^2 + x_3 = 5$

$5x_1^2 - x_2^2 - x_3 \geq 0$

$x_1, x_2, x_3 \geq 0.$

D–2200

Sub. Code
31133

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is meant by divisibility? Give an example.

2. Define the Mobius function $\mu(n)$.

3. Prove that $\phi(m,n) = \phi(m) \,.\, \phi(n)$ if $(m, n) = 1$.

4. Define Liouville's function $\lambda(n)$.

5. State the Little Fermat theorem.

6. Prove that $[x + n] = [x] + n$.

7. What are the solutions of the congruence $x^2 \equiv 1 (\bmod 8)$?

8. Write down the Legendre's symbol (n/p).

9. What are the quadratic residues and non-residues mod 13?

10. Define the Jacobi symbol.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

Or

(b) State and prove Euclidean algorithm.

12. (a) Prove that $d(n)$ is odd if and only if n is a square.

Or

(b) Let f be multiplicative. Prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

13. (a) State and prove Euler's summation formula.

Or

(b) Prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + 0\left(\frac{1}{x}\right)$, if $x \geq 1$.

14. (a) Prove that $5n^3 + 7n^2 \equiv 0 (\text{mod} 12)$ for all integer n.

Or

(b) State and prove Chinese remainder theorem.

15. (a) Determine whether 219 is a quadratic residue or non-residue mod 383.

Or

(b) State and prove reciprocity law for Jacobi symbol.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

17. State the Mobius inversion formula. Also derive the Selberg identify.

18. Derive Dirichets' asymptotic formula.

19. State and prove Wilson's theorem.

20. Prove that the diaphantine equation $y^2 = x^3 + K$ has no solution if K has the form $K = (4n-1)^2 - 4m^2$ where m and n are integers such that no prime $p \equiv -1 \pmod 4$ divides m.

D–2201

Sub. Code
31134

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Markov chain. Give an example.
2. Define transition probability.
3. What is meant by class property?
4. Write the forward diffusion equation of the Wiener process.
5. Define sample paths.
6. What is Ornstein – Uhlenbeck process?
7. When does a Galton Watson process is a Markov chain?
8. Define a Markov branching process.
9. Explain inter arrival time.
10. What is meant by birth and death process?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Derive the mean recurrence time for the state j as $\mu_{jj} = \sum_{n=1}^{\infty} n f_{jj}^{(n)}$.

Or

(b) State and explain the postulates for Poisson process.

12. (a) If $\{N(t)\}$ is a Poisson process and $s < t$ then prove that $P_r\{N(s) = K \mid N(t) = n\} = \binom{n}{k} (s/t)^K (1-(s/t))^{K-1}$.

Or

(b) If $X(t)$, with $X(0)$ and $\mu = 0$, is a Wiener process, show that $Y(t) = \sigma X\left(t/\sigma^2\right)$ is also a Wiener process. Find its covariance function.

13. (a) If $m \leq 1$, the probability of ultimate extinction is 1. If $m > 1$, the probability of ultimate extinction is the positive root less than unity of the equation $s = p(s)$ – prove.

Or

(b) Show that the p.d.f. of one of the conditional distribution of X_n, given $X_n > 0$ is $\frac{P_n(s) - P_n(0)}{1 - P_n(0)}$. Find $P_r\{X_n = r \mid X_n > 0\}$. When P_r {number of offspring = K} = $\left(\frac{1}{2}\right)^{K+1}$, $K =$ 0, 1, 2....

14. (a) If $m = E(X_1) = \sum_{K=0}^{\infty} Kp_k$ and $\sigma^2 = \text{var}(X_1)$ then prove that $E\{X_n\} = m^n$ and

$$\text{var}(X_n) = \begin{cases} \dfrac{m^{n-1}(m^n - 1)}{m-1}\sigma^2, & if\ m \neq 1 \\ n\sigma^2, & if\ m = 1 \end{cases}.$$

Or

(b) Prove that the p.g.f. $R_n(s)$ of Y_n satisfies the recurrence relation $R_n(s) = sP(R_{n-1}(s))$, $P(s)$ being the p.g.f. of the offspring distribution.

15. (a) Explain about steady state distribution.

Or

(b) Explain the model $M/M/\infty$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the Chapman – Kolmogorov equation.

17. Prove that state j is persistent if and only if $\sum_{n=0}^{\infty} P_{jj}^{(n)} = \infty$.

18. Prove that the generating function $F(t, s) = \sum_{K=0}^{\infty} P_r\{X(t) = k\} s^K$ of an age-dependent branching process $\{X(t), t \geq 0\}$; $X_0 = 1$ satisfies the integral equation $F(t,s) = [1 - G(t)] + \int_0^t P(F(t-u, s))dG(u)$.

19. Prove that $P_n(s) = P_{n-1}(P(s))$ and $P_n(s) = P(P_{n-1}(s))$ where $P_n(s) = \sum_K P_K s^K$.

20. Derive the Erlang's second formula $C(s, \lambda/\mu) = \frac{P_s}{1-\rho}$.

D–2202

Sub. Code
31141

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a spanning subgraph of a graph *G*.

2. Define cut vertex of a graph. Give an example.

3. Define a path a cycle.

4. Define cut vertex and bridge of a graph.

5. Define perfect matching.

6. What is meant by covering number of a graph *G?*

7. Define the chromatic number of a graph.

8. Define planar graph.

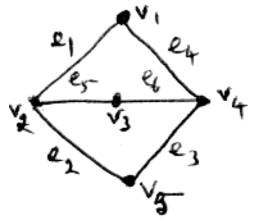
9. State the five colour theorem.

10. Define directed cycle with an example.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define incidence matrix of a graph. Also find the incidence matrix of the graph :



Or

(b) Prove that every tree has a centre consisting of either one vertex or two adjacent vertices.

12. (a) For what values of n does the complete graph K_n have perfect matching?

Or

(b) Prove that $\gamma(m,n) = \gamma(n,m)$.

13. (a) If G is uniquely n-colourable, then prove that $\delta(G) \geq n-1$.

Or

(b) Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

14. (a) Prove that if e is a bridge of a connected graph G, then $G-e$ has exactly two components.

Or

(b) Show that there is not map of five regions in the plane such that every pair of regions are adjacent.

15. (a) If two diagraphs are isomorphic then prove that the corresponding vertices have the same degree pair.

Or

(b) State and prove max-flow, min-cut theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let G be a (p,q) graph. Prove that the following are equivalent.

(a) G is a tree.

(b) Every two vertices of G are joined by a unique path.

(c) G is connected and $p = q + 1$.

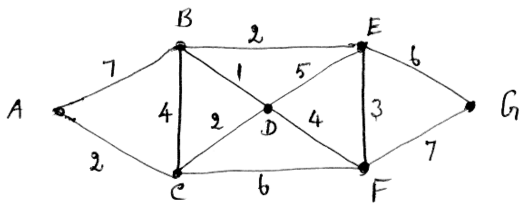
(d) G is acyclic and $p = q + 1$.

17. State and prove that Chavatal theorem.

18. State and prove Brook's theorem.

19. If G is a connected plane graph having V, E and F as the sets of vertices, edges and faces respectively, then prove that $|V| - |E| + |F| = 2$.

20. Find the shortest distance of the vertex G from the vertex A of the following weighted graph.



D–2203

Sub. Code
31142

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define bounded linear map.

2. Define banach Space. Give an example.

3. Define completely continuous map.

4. Define inner product space. Give an example.

5. What is meant by annihilator?

6. Define projection mapping.

7. Define Hilbert space.

8. What is meant by unitary operator?

9. State the Riez theorem.

10. State closed graph theorem.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a normed space. If E_1 is open in X, and $E_2 \subset X$, then prove that $E_1 + E_2$ is open in X.

Or

(b) Prove that l_∞ is a complete space.

12. (a) Let X and Y be normed spaces and $F : X \to Y$ be a linear map such that the rang $R(F)$ of F is finite dimensional. Prove that F is continuous if and only if the zero space $Z(F)$ of F is closed in X.

Or

(b) Prove that a Banach space cannot have denumerable basis.

13. (a) Prove that a closed subspace of a compact space is compact.

Or

(b) Let X and Y be normed space and let $F : X \to Y$ be linear. Prove that f is continuous if and only if $g\,o\,f$ is continuous for every $g \in Y'$.

14. (a) Let X be an inner product space and $f \in X'$. Let $\{v_\alpha\}$ be an orthonormal set in X and $E_f = \{u_\alpha : f(u_\alpha) \neq 0\}$. Then E_f is a countable set say $\{u_1, u_2 ....\}$. If E_f is denumerable, then prove that $f(u_n) \to 0$ as $n \to \infty$.

Or

(b) State and prove Bessel's inequality.

15. (a) Define strong convergence. Show that weak convergence does not imply strong convergence.

Or

(b) If X is a finite dimensional normed space, prove that strong convergent is equivalent to weak convergence.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that a subset A of a metric space (X,d) is sequentially compact if and only if it is compact.

17. Let X be a normed space. Prove that the following are equivalent.

(a) Every closed and bounded subset of X is compact.

(b) The subset $\{x \in X : \|x\| \leq 1\}$ of X is compact.

(c) X is finite dimensional.

18. State and prove Hahn Banach extension theorem.

19. Let H be a Hilbert space, G be a subspace of H and g be a continuous linear functional on G. Prove that there is a unique continuous linear functional f on H such that $f \mid g = g$ and $\|f\| = \|g\|$.

20. State and prove uniform boundedness theorem.

D–2204

Sub. Code
31143

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Write the sufficient condition on $\phi(x)$ for convergence in iteration methods.

2. Explain sturm sequence.

3. Write the deflated polynomial.

4. Define an eigen values of a matrix A.

5. What is the truncation error of the Lagrange quadratic interpolating polynomial?

6. What is meant by Knots?

7. What is spline fitting?

8. Write the formula for Gauss method.

9. Write the formula for second order $R-K$ method.

10. What are the merits and demerits of Taylor's method?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find a root of $\cos x - x^2 - x = 0$ to five decimal places.

Or

(b) Use synthetic division and perform two iterations of the Birge – Vieta method to find the smallest positive root of $P_3(x) = 2x^3 - 5x + 1 = 0$.

12. (a) Determine the condition number of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ using the maximum absolute row sum norm.

Or

(b) Prove that no eigen value of a matrix A exceeds the norm of a matrix. (ie, $\|A\| \geq \rho(A)$).

13. (a) Derive the Hermite interpolating polynomial.

Or

(b) Using the following values of $f(x)$ and $f'(x)$, estimate the values of $f(-0.5)$ and $f(0.5)$ using piecewise cubic Hermite interpolation.

14. (a) Obtain the least squares polynomial approximation of degree one and two for $f(x) = x^{1/2}$ on [0, 1].

Or

(b) The following table of values is given :

x	–1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

Using the formula $f'(x_1) = (f(x_2) - f(x_0))/(2h)$ and the Richardson extrapolation, find $f'(3)$.

15. (a) Discretize the initial value problem $y' = 1 + y,\ y(1) = 0$ using backward Euler method. Compute $y(1.2)$ using $h = 0.1$.

Or

(b) Use the Taylor series method of order four to solve the initial value problem $u' = t^2 + u^2$, $u(0) = 1$ for the interval (0, 0.4) using two subintervals of length 0.2.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $P_3(x) = x^3 + x^2 - x + 2 = 0$. Use the initial approximations $p_0 = -0.9, q_0 = 0.9$.

17. Using Jacobi method, find all the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

18. Construct the bivariate interpolating polynomial and hence find $f(0.5, 0.5)$ from the following function $f(x, y)$:

y \ x	0	1	3
0	1	2	10
1	2	4	14
3	10	14	28

19. Evaluate the integral $\int_0^2 e^x dx$ using Simpson's rule with $h = 1$ and 1/2. Find a bound on the error in each case. Compare with exact solution.

20. By applying fourth order $R - K$ method find $y(0.2)$ from $y' = y - x$, $y(0) = 2$ taking $h = 0.1$.

D–2205

Sub. Code
31144

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2023.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours — Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. If the sample space is $\mathscr{C} = c_1 \cup c_2$ and if $P(c_1) = 0.8$ and $P(c_2) = 0.5$, then find $P(c_1 \cap c_2)$.

2. Define the distribution function of X and Y.

3. Prove that $E[(X - \mu_1)(Y - \mu_2)] = E(XY) - \mu_1\mu_2$.

4. Let the joint p.d.f. of X_1 and X_2 be $f(x_1, x_2) = \begin{cases} 12 \quad x_1 x_2 (1 - x_2), & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$. Are x_1 and x_2 are stochastically independent? Justify.

5. Determine the binomial distribution for which the mean is 4 and variance is 3.

6. Write the p.d.f. of gamma distribution.

7. Define t-distribution.

8. What do you meant by the change of variable technique?

9. Define convergence in distribution.

10. State the central limit theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If X has the p.d.f. $f(x) = \begin{cases} (1/2)^x, & x = 1,2,3.... \\ 0, & \text{elsewhere} \end{cases}$ find the m.g.f, mean and variance of X.

Or

(b) Let X have the p.d.f. $f(x) = \begin{cases} (x+2)/18, & -2 < x < 4 \\ 0, & \text{elsewhere} \end{cases}$. Find $E(X)$, $E\left[(x+2)^3\right]$ and $E\left[6x - 2(x+2)^3\right]$.

12. (a) Find $P_r\left(0 < X_1 < \frac{1}{3}, 0 < X_2 < 1/3\right)$ if the random variables X_1 and X_2 have the joint p.d.f. $f(x) = 4x_1(1 - x_2)$, $0 < x_1 < 1$, $0 < x_2 < 1$; zero elsewhere.

Or

(b) Compute the measure of Skewness and Kurtosis of the binomial distribution $b(n, p)$.

13. (a) Find the mean and variance of Poisson distribution.

Or

(b) Let X have the p.d.f. $f(x) = \begin{cases} (1/2)^x, & x = 1,2,3.... \\ 0, & \text{elsewhere} \end{cases}$. Find the p.d.f. of $Y = X^3$.

14. (a) Let X_1, X_2 be a random sample from a distribution having the p.d.f. $f(x) = \begin{cases} e^{-x}, & 0 < x < a \\ 0, & \text{elsewhere} \end{cases}$. Show that $Z = X_1/X_2$ has a F-distribution.

Or

(b) Derive the p.d.f. of chi-square distribution.

15. (a) Let z_n be $\chi^2(n)$ and let $w_n = \frac{z_n}{n^2}$, find the limiting distribution of W_n.

Or

(b) Let $\overline{X}_n$ denote the mean of the random variable of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, determine a lower bound for the probability $P_r(-2 < x < 8)$ using Chebyshev inequality.

17. Find the moment generating function, mean and variance of the gamma distribution.

18. Derive t – distribution and derive the p.d.f. of the t-distribution.

19. Let $T = \frac{W}{\sqrt{V/\gamma}}$ where W and V are respectively normal with mean 0 and variance 1. Chi-square with γ. Show that T^2 has an F-distribution with parameters $\gamma_1 = 1, \gamma_2 = \gamma$.

20. State and prove the central limit theorem.

D–1452

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, MAY 2019.

First Semester

Mathematics

ALGEBRA — I

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. If G is a group, then show that the identify element of G is unique.

2. Let G be the group of integers under addition and H be the set of all multiples of 3. Find $i_G(H)$.

3. Show that every subgroup of an abelian group is normal.

4. Compute $a^{-1}ba$ where $a = (5,7,9)$ $b = (1,2,3)$.

5. If G is a group and H is a subgroup of index 2 in G, prove that H is a normal subgroup of G.

6. Give an example of a commulative ring has no unit element.

7. If $a,b \in R$, R is a ring, evaluate $(a+b)^2$.

8. If U is an ideal of a ring R and $1 \in U$, prove that $U = R$.

9. Find all the units in $J[i]$.

10. If $a|b$, then prove that $a|bx$ for all $x \in R$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that a non empty subset H of the group G is a subgroup of G if and only if $ab^{-1} \in H$, for all $a, b \in H$.

Or

(b) If H and K are finite subgroups of G of order $O(H)$ and $O(K)$ respectively, then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$.

12. (a) If G is a group, N a normal subgroup of G, then prove that G/N is also a group.

Or

(b) State and prove Cayley theorem.

13. (a) If $O(G) = p^2$, where p is a prime number, then prove that G is abelian.

Or

(b) If U is an ideal of the ring R, then prove that R/U is a ring and is a homomorphic image of R.

14. (a) Let R be a commutative ring with unit element whose only ideals are (o) and R itself. Then prove that R is a field.

Or

(b) Prove that every Euclidean ring possess a unit element.

15. (a) State and prove Unique factorization theorem.

Or

(b) State and prove the Einstein criterion.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that the number of conjugate classes in S_n is $p(n)$, the number of position of n.

17. Prove that the number of p-sylow subgroups in G is of the form $1 + 19p$.

18. Show that every finite abelian group is the direct product of cyclic group.

19. Prove that every integral domain can be imbedded in a field.

20. Prove that if R is a unique factorization domain, then so is $R[2]$.

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D–1453

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2019.

First Semester

ANALYSIS - I

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. If a and b are real, then show that $(a, b) = a + bi$.

2. If $x > 0$ and $y < z$, then prove that $xy < xz$.

3. Construct a bounded set of real numbers with exactly 5 limit points.

4. Is $d(x, y) = |x - 2y|$, metric on $\mathbb{R}$? Justify.

5. Calculate $\lim_{n \to \infty} \left(\sqrt{n^2 + n} - n\right)$.

6. If $\sum a_n$ converges, then prove that $\lim_{n \to \infty} a_n = 0$.

7. Let E be a non-compact set in $\mathbb{R}$. Then show that three is a continuous function on E which is not bounded.

8. Let f be a continuous real function on a metric space X. Let $Z(f) = \{ p \in X / f(p) = 0\}$, prove that $z(f)$ in closed.

9. What are the adherent points of $A = (0,1] \cup \{2\}$?

10. Let f: $\mathbb{R} \to \mathbb{R}$ defined by $f(x) = x$? Find the derivative of f at x.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Let $A = \{p \in Q / p^2 < 2\}$ and $B = \{p \in Q / p^2 > 2\}$. Show that A contains no largest number and B contains no smallest number.

Or

(b) If $x \in \mathbb{R}$, $y \in \mathbb{R}$ and $x < y$, then show that there is a $p \in Q$ such that $x < p < y$.

12. (a) Prove that the compact subsets of metric spaces and closed.

Or

(b) What are the connected subsets of $\mathbb{R}^1$? Justify your answer.

13. (a) Prove that the sub sequential limits of a sequence in a metric space X form a closed subset of X.

Or

(b) For any sequence $\{a_n\}$ of positive numbers, show that $\lim\limits_{n \to \infty} \sup \sqrt[n]{a_n} \leq \lim\limits_{n \to \infty} \sup \frac{a_{n+1}}{a_n}$.

14. (a) Prove that a map $f: X \to Y$ is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

Or

(b) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then show that f is uniformly continuous on X.

15. (a) State and prove generalized mean value theorem.

Or

(b) Let f be defined for all real x, and suppose that $|f(x)-f(y)| \leq (x-y)^2$ for all real x and y prove that f is a constant function.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Show that every k - cell is compact.

17. Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

18. Let A and B be disjoint non-empty closed sets in a metric space X, define $f: X \to \mathbb{R}$ by $f(x) = \frac{P_A(x)}{P_A(x) + P_B(x)}$, $x \in X$ where $P_A(x) = \inf d(x, y)$, $y \in A$.

Show that f in a continuous function on X, $f(x) \subseteq [0, 1]$, $f(x) = 0$ precisely on A and $f(x) = 1$ precisely on B.

19. State and prove cantor intersection theorem.

20. State and prove the inverse function theorem.

D–1454

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2019.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2018-19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Find all solutions of $y'' + y' - 2y = 0$.

2. Find the solutions of the initial value problem $y'' + 10y = 0,\ y(0) = \pi,\ y'(0) = \pi^2$.

3. Define linearly dependent function.

4. Find the linearly independent solution of $y'' - y' - 2y = e^{-x}$.

5. Define analytic function.

6. Define regular singular point.

7. Write the Bessel function of zero order of the first kind.

8. Write the n^{th} Legendre polynomial.

9. Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y^1 = y^2, y(0) = 2$.

10. Find the value of $p_n(1)$ and $p_n(-1)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find all solutions of $y'' - 4y' + 5y = 3e^{-x} + 2x^2$.

Or

(b) Solve : $y'' + (3i - 1)y' - 3\,iy = 0$, $y(0) = 2$, $y'(0) = 2$.

12. (a) One solution of $x^2y'' - xy' + y = 0$, $(x > 0)$ is $\phi_1(x) = x$ find the system ψ of $x^2y'' - xy' + y = e^x$ satisfying $\psi(1) = 1, \psi'(1) = 0$.

Or

(b) Compute three linearly independent solutions of $y''' - 4y' = 0$.

13. (a) Solve : $y'' - 3y' + 2y = x^2$.

Or

(b) Verify that the function $\phi_1(x) = x, (x > 0)$ satisfies the equation $x^2y'' - xy' + y = 0$, and find a second independent solution.

14. (a) Find two linearly independent power series solutions of $y'' + 3x^2 y' - xy = 0$.

Or

(b) Prove that between any two positive zeros of J_0 there is a zero of z_1.

15. (a) Let f be a real-valued continuous function on the strip $S : |x| \le \alpha$, $|y| < \infty$ $(\alpha > 0)$ and suppose f satisfies a Lipschitz condition on S. Show that the solution of the initial value problem is $y'' + \lambda^2 y = f(x, y), y(0) = 0$, $y'(0) = 1$, $(\lambda > 0)$ is unique.

Or

(b) Show that the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n(\underline{|n})^2}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If ϕ_1 is a solution of $L(y)[y'' + a_1(x)\, y' + a_2(x)\, y] = 0$ on an interval I, and $\phi_1(x) \neq 0$ on I, prove that a second solution ϕ_2 of $L(y) = 0$ is $\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \frac{1}{[\phi(s)]^2}$

$\exp\left[- \int_{x_0}^{x} a_1(t)\, dt \right] ds$. Also prove that the function ϕ_1, ϕ_2 form a basis for the solutions of $L(y) = 0$ on I.

17. Prove that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I ,if and only if, $W(\phi_1, \phi_2)(x) \neq 0$.

18. Derive Bessel's function of zero order of the second kind K_0.

19. Find all real valued solutions of

(a) $y' = x^2 y$

(b) $y' = \dfrac{e^x - y}{1 + e^x}$.

20. Find a solution ϕ of $y'' = \dfrac{-1}{2y^2}$ satisfying $\phi(0) = 1, \phi'(0) = -1$.

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D–1455

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2019.

First Semester

TOPOLOGY — I

(CBCS 2018-2019 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define onto function.

2. Define countably infinite set. Give an example.

3. State the maximum principle theorem.

4. What is meant by subspace topology?

5. Define metric topology.

6. State intermediate value theorem.

7. Define compact space. Give an example.

8. State finite intersection property.

9. Define second countable space. Give an example.

10. Prove that the product space $S_\Omega \times \overline{S}_\Omega$ is completely regular.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) State and prove strong induction principle theorem.

Or

(b) Show that the set of all rational numbers Q is countably infinite.

12. (a) Let X be a set; let $\mathcal{B}$ be a basis for a topology τ on X. Prove that τ equals the collection of all various of elements of $\mathcal{B}$.

Or

(b) State and prove sequence lemma.

13. (a) Prove that every finite point set in a Hausdorff space X is closed.

Or

(b) Prove that the image of a connected space under a continuous map is connected.

14. (a) Prove that a finite union of compact subspaces of a topological space X is compact.

Or

(b) Prove that a finite Cartesian product of connected space is connected.

15. (a) State and prove the Lebesgue number lemma.

Or

(b) Show that every regular Lindelof space is normal.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If C is an infinite subset of Z_+, then prove that C is countably infinite.

17. Let $f: X \to Y$; let X and Y be metrizable with metrics d_x and d_y, respectively. Prove that the continuity of f is equivalent to the requirement that given $x \in X$ and given $\notin > 0$, there exists $\delta > 0$ such that $d_x(x, y) < \delta \Rightarrow d_y(f(x), f(y)) < \in$

18. Prove that the product of finitely many compact space is compact.

19. State and prove Urysohn lemma.

20. If L is a linear continuum in the order topology then prove that L is connected, and so are intervals and rays in L.

D-1456

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2019.

Second Semester

ALGEBRA — II

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define linear span.

2. Define orthonormal set in an inner product space.

3. State remainder theorem.

4. Define inner product space.

5. If A and B are finite dimensional subspaces of a vector space V, then prove that
$\dim(A+B) = \dim(A) + \dim(B) - \dim(A \cap B)$.

6. Define splitting field.

7. Prove that the fixed field of G is a sub field of K.

8. Find the Galois group $G(C:R)$.

9. Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

10. If $T \in A(V)$ is Hermitian, show that all its characteristic roots are real.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If V is a finite dimensional vector space over F and $u_1, u_2, ..., u_n$ span V then show that some subset of $u_1, u_2, ..., u_n$ forms a basis of V.

Or

(b) Prove that $A(A(W)) = A(W)$.

12. (a) If V is finite dimensional and W is a subspace of V, then prove that $\hat{W}$ is isomorphic to $\hat{V} / A(W)$ and $\dim A(W) = \dim V - \dim W$.

Or

(b) If V is a finite dimensional inner product space and W is a subspace of V, prove that $V = W + W^{\perp}$.

13. (a) Prove that a polynomial of degree n can have atmost n roots in any field exfension.

Or

(b) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

14. (a) Define normal extension and show that, if K is a normal extension of F then K is a splitting field of some polynomial over F .

Or

(b) Prove that $G(K, F)$ is a subgroup of the group of all automorphisms of K.

15. (a) If $\langle VT, VT\rangle = \langle V, V\rangle$ for all $V \in V$, then prove that T is unitary.

Or

(b) If V is finite dimensional over F. Then show that $T \in A(V)$ is singular if and only if there exists a $av \neq 0$ in V such that $T(V) = 0$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that if V is a finite dimension vector space over a field F and W is a subspace of V, then W is finite dimensional, $\dim W \leq \dim V$ and

$$div\left[\frac{V}{W}\right] = \dim(V) - \dim(W) .$$

17. Let V be a finite dimensional inner product space, then prove that V has an orthonormal set as a baiss.

18. Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

19. State and prove the fundamental theorem of Galosis theory.

20. If $\lambda \in F$ is a characteristic root of $T \in A(V)$, shw that for every $q(x) \in F[x], q(\lambda)$ is characteristic root of $q(x)$. Using this prove that λ is a root of minimal polynomial of T.

D–1457

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, MAY 2019.

Second Semester

Mathematics

ANALYSIS — II

(CBCS — 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Let $f, \alpha : [1, 5] \to \mathbb{R}$ defined by $f(x) = \begin{cases} 1 & if \quad x \in [1,5] \cap Q \\ 0 & if \quad x \in [1, 5] \cap Q^c \end{cases}$ and $\alpha(x) = x \forall x \in [1, 5]$, p be any partition of $[1, 5]$. Find $U(p, f, \alpha)$ and $L(p, f, \alpha)$.

2. Define rectifiable curve.

3. For $m = 1, 2, 3..., n = 1, 2, 3...$ let $s_{m,n} = \frac{m}{m+n}$. Find $\underset{n\to\infty}{lt}\ \underset{m\to\infty}{lt}\ s_{m,n}$.

4. Define Equicontinuous families of functions.

5. Prove that $\Gamma(x+1) = x\Gamma(x)$.

6. Find $\lim_{x \to 0} \frac{x - \sin x}{\tan x - x}$.

7. Show that $\chi_{A\cap B} = \chi_A \cdot \chi_B$.

8. If A is countable, prove that $m^*(A) = 0$.

9. Show that if f is integrable over E, $\left|\int_E f\right| \leq \int_E |f|$.

10. Prove that $|f| = f^+ + f^-$.

PART B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Prove that $f \in \mathbb{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition p such that $U(p, f, \alpha) - L(p, f, \alpha) < E$.

Or

(b) State and prove the fundamental theorem of Calculus.

12. (a) If $\{f_n\}$ is a sequence of continuous functions on E , and if $f_n \to f$ uniformly on E , then show that f is continuous on E .

Or

(b) Prove that there exists a real continuous function on $\mathbb{R}$ which is no-where differentiable.

13. (a) Suppose Σc_n converges. Let $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $-1 < x < 1$. Then show that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

Or

(b) If f is continuous (with period 2π) and if $\varepsilon > 0$, prove that there is a trignometric polynomial p such that $|p(x) - f(x)| < \varepsilon$.

14. (a) Prove that any interval is measurable.

Or

(b) Show that the family of measurable sets is σ – algebra.

15. (a) Let f be a bounded function defined on $[a, b]$. If f is Reimann integrable on $[a, b]$, then show that f is measurable and $R\int_a^b f(x)dx = \int_a^b f(x)\,dx$.

Or

(b) State and prove Lebesgue Dominated Convergence theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If γ' is continuous on $[a, b]$ then show that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\,dt$.

17. If K is compact, $f_n \in \mathscr{C}(k)$ and $\{f_n\}$ is pointwise bounded and equi continuous on k, then show that

(a) $\{f_n\}$ is uniformly bounded on k

(b) $\{f_n\}$ contains a uniformly convergent subsequence.

18. State and prove Parseval's theorem.

19. Construct a non-measurable subset of $\mathbb{R}$.

20. (a) State and prove Fatou's lemma.

    (b) State and prove Monotone Convergence theorem.

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D–1458

Sub. Code
31123

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2019.

Second Semester

TOPOLOGY – II

(CBCS – 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define one point compactification. Give an example.

2. Define lcoally finite.

3. Explain the term Stone-cěch compactification.

4. Is the space $(-1, 1)$ in R complete? Justify.

5. Show that the real line R is neither bounded nor totally bounded under the metric $d(x, y) = |x - y|$.

6. What is Compact-open topology?

7. Define compactly generated space.

8. Let $A \subset X$; let $f : A \to Z$ is a continuous map of A into the Hausdorff space Z. Show that there is atmost one extension of f to a continuous function $g : \overline{A} \to Z$.

9. Define locally metrizable space.

10. Define Baire space.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Let X be a Hausdorff space. Prove that X is locally compact if and only if given x in X, and given a neighborhood U of X, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$.

Or

(b) Prove that a product of completely regular space is completely regular.

12. (a) Under what conditions does a metrizable space have a metrizable compactification?

Or

(b) Show that the collection $B = \left\{ \left(0, \frac{1}{n}\right) \Big/ n \in Z_{+} \right\}$ is locally finite in (0, 1) but not in R.

13. (a) Prove that every closed subspace of a paracompact space is paracompost.

Or

(b) Prove that every regular Lindelof space is paracompact.

14. (a) Prove that a metric space X is complete if every Cauchy sequence in X has a convergent subsequence.

Or

(b) If the space Y is complete in the metric d, then show that the space Y^J is complete in the uniform metric $\overline{\rho}$ corresponding to d.

15. (a) If X is a compact Hausdorff space or a complete metric space, then prove that X is a Baire space.

Or

(b) Show that the topologist's sine curve has dimension 1.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tychonoff theorem.

17. Prove that a space X is metrizable if and only if X is regular and has a basis that is countably locally finite.

18. Let (X, d) has a metric space. Prove that there is an isometric imbedding of X into a complete metric space.

19. Let $I = [0, 1]$. Prove that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2.

20. Prove that every compact space of $\mathbb{R}^N$ has topological dimension at most N.

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D–1459

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, MAY 2019.

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 19 Academic year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define orthogonal trajectory.

2. Show that the equation $2xzdx + z\,dy - dz = 0$ is not integrable.

3. Define the order of a partial differential equation.

4. Write two non-linear partial differential equations.

5. Define singular integral.

6. Find the complete integral of $p + q = pq$.

7. Find the a particular integral of $\left(D^2 - D'\right)z = e^{x+y}$.

8. State the interior Neumann problem.

9. Form the partial differential equation by eliminating f from $z = f(x-y)$.

10. Classify the one dimensional diffusion equation

$$\frac{\partial^2 z}{\partial x^2} d = \frac{\partial z}{\partial y}.$$

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Find the integral curves of the sets of equations.

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}.$$

Or

(b) Determine the integrability of the equation

$$z(z+y)\,dx + z(z+x)\,dy - 2xydz = 0.$$

12. (a) Form the partial differential equation by eliminating arbitrary function f from $f(x^2+y^2+z^2, z^2-2xy) = 0$.

Or

(b) Find the general solution of the differential equation.

$$x^2 \frac{\partial z}{\partial z} + y^2 \frac{\partial z}{\partial y} = (x+y)z.$$

13. (a) Solve $(x-2z)\,p + (2z-y)\,q = y - x$.

Or

(b) Show that the equations $xp = yq$ and $z(xp+yq) = 2xy$.

14. (a) Find the complete integral of

$$pqz = p^2(xq + p^2) + q^2(yp + q^2)$$

Or

(b) Using Jacobi's method, solve $2(y + zq) = q(xp + yq)$.

15. (a) Find a particular integral of the equation $(D^2 - D')z = e^{x+y}$.

Or

(b) Find the D'Alembert's solution of the one dimensional wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.

17. Find the general integral of the linear partial differential equation $px(x + y) = qy(x + y) - (x - y)(2x + 2y + z)$.

18. Find the equation of the integral surface of the differential equation $2y(z-3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0$ and $x^2 + y^2 = 2x$.

19. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

20. A uniform string of line density ρ is stretched to tension ρc^2 and executes a small transverse vibration in a plane through the undistributed line of the string. The ends

$x = 0$ and $x = l$ of the string are fixed. The string is at rest, with the point $x = b$ drawn aside through a small distance E and relased at time $t = 0$. Show that any subsequent time t the transverse displacement y is given by the Fourier series $y = \frac{2 \in l^2}{\pi^2 b(l-b)} \sum_{s=1}^{\infty} \frac{1}{s^2} \sin\frac{s\pi b}{l}$ $\sin\frac{s\pi x}{l} \cos\frac{s\pi ct}{l}$.

D–6871

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

First Semester

ALGEBRA — I

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define onto mapping.

2. If G is a group, show that every $a \in G$ has a unique inverse in G.

3. Define order of an element.

4. Define normalizer of an element in a group.

5. State the pigeonhole principle.

6. Give an example of an integral domain which has an infinite number of elements.

7. Show that (4) is not a prime ideal in $\mathbb{Z}$.

8. Define Euclidean ring.

9. Prove that 5 is not prime element in the ring R of Gaussian integers.

10. Let R be an Euclidean domain. Suppose that $a, b, c, \in R$, a / bc but $(a, b) = 1$ prove that a / c.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any two sets A and B, prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

Or

(b) Prove that HK is a subgroup of G if and only if $HK = KH$

12. (a) Show that any two p-sylow subgroups of a group G are conjugate.

Or

(b) State and prove Cauchy's theorem for abelian group.

13. (a) Prove that a ring homorphism $\phi : R \to R'$ is one to one if and only if the Kernal of ϕ is zero submodule.

Or

(b) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R / M is a field.

14. (a) Let R be an Euclidean ring. Suppose that for $a, b, c \in R$, a / bc but $(a, b) = 1$, then prove that a / c.

Or

(b) If R is an Euclidean domain prove that any two elements a and b in R have greatest common divisor.

15. (a) Let $f(x)$, $g(x)$ be two non-zero elements in $F[x]$. Prove that $\deg f(x) \leq \deg(f(x) \cdot g(x))$.

Or

(b) State and prove the division algorithm.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$ respectively, then prove that $o(HK) = \frac{o(H)\, o(K)}{o(H \cap K)}$.

17. For a given prime p, show that the number of p-sylow subgroups of G is of the form $1 + kp$.

18. Let $R = \{(a, b) / a, b \in R\}$ and the operator addition and multiplication are defined as $(a,b) + (c,d) = (a + c, b + d)$ and $(a, b) \cdot (c,d) = (ac - bd, ad + bc)$. Show that R is a field.

19. State and prove Gauss lemma.

20. Prove that if R is a unique factorization domain then $R[x]$ is also unique factorization domain.

D–6872

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

First Semester

ANALYSIS – I

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Balls and convex – Justify.

2. Prove that a set E is open if its complement is closed.

3. Define compact set. Give an example.

4. Define convergence of a sequence.

5. Define informally continuous function. Give an example.

6. Write short notes on rearrangement of $\sum a_n$.

7. Define discontinuity of second kind.

8. Define derived set.

9. State the intermediate value theorem.

10. State the Rolle's theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Let z and w be complex numbers. prove that $|z+w| \leq |z|+|w|$.

Or

(b) Prove that the compact subsets of a metric spaces are closed.

12. (a) Prove that the subsequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of xX

Or

(b) Calculate $\lim_{n\to\infty}(\sqrt{n^2+n}-n)$

13. (a) Prove that a mapping f of a metric space x into a metric space Y is continuous if and only if $f^{-1}(v)$ is open in X for every open set v in y.

Or

(b) Prove that monotomic functions have no discontinuities of the second kind.

14. (a) If f is a continuous mapping of a metric space X into a metric space Y and if E is connected subset of X prove that $f(E)$ is connected.

Or

(b) Prove that the intersection of a finite collection of open set is open.

15. (a) State and prove generalized mean value theorem.

Or

(b) If f is differentiable at c, then prove that f is continuous at c.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

16. Prove that every K-cell is compact.

17. Prove that $\Sigma \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

18. State and prove the Heine-Borel covering theorem.

19. State and prove Taylor's theorem.

20. State and prove the inverse function theorem.

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D–6873

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the existence theorem for linear equations with constant coefficients $L(y) = 0$.

2. Define Wronskian.

3. Write any two Legendre polynomials.

4. Find the regular singular points of $(1+x^2)\,y'' - 2xy' + 2y = 0$.

5. Define Indicial polynomial.

6. Determine the equation $2xy\,dx + (x^2 + 3y^2)\,dy = 0$ is exact or not.

7. Show that the solution ϕ of $y' = y^2$ which passes through the point (x_0, y_0) is given by $\phi(x) = \dfrac{y_0}{1 - y_0(x - y_0)}$.

8. Find all real valued solution of the equation $y' = x^2 y$.

9. State the local existence theorem for initial value problem $y' = f(x, y),\ y(x_0) = y_0$.

10. Consider the initial value problem

$y_1' = y_2^2 + 1$

$y_2' = y_1^2$

$y_1(0) = 0,\ y_2(0) = 0.$

Compute the first three successive approximation ϕ_0, ϕ_1, ϕ_2.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that there exist n linearly-independent solutions of $L(y) = 0$ on I.

Or

(b) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0, then prove that

$W(\phi_1, \phi_2)(x) = e^{-\alpha_1 (x - x_0)} W(\phi_1, \phi_2)(x_0)$.

12. (a) Find all solutions of the equation

$x^2 y''' + 2x^2 y'' - xy' + y = 0,$ for $x > 0$.

Or

(b) Show that $\int_{-1}^{1} P_n^2(x)\,dx = \dfrac{2}{2n+1}$.

13. (a) One solution of $y'' + \frac{2}{x^2} y = 0$ is $\phi_1(x) = x^2, 0 < x < \infty$.

Find all solutions of $y'' - \frac{2}{x^2} y = x, 0 < x < \infty$.

Or

(b) Find two linearly independent power series solutions of $y'' - xy' + y = 0$.

14. (a) Show that $x^r = e^{i\pi r}|x|^r, x < 0$, where $x^r = e^{r \log x}$, for $x > 0$ and for $x < 0$.

Or

(b) Find the singular points of $x^2 y'' + (x + x^2) y' - y = 0$ and determine those which are regular singular points.

15. (a) Find all real valued solutions of $y' = x^2 y^2 - 4x^2$.

Or

(b) By computing appropriate Lipschitz constants, show that $f(x, y) = 4x^2 + y^2$, on $S : |x| \le 1, |y| \le 1$ satisfy Lipschitz conditions on S.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Solve :

(a) $y'' + 4y = \cos x$

(b) $y'' - 7y' + 6y = \sin x$.

17. Find all solutions of

(a) $y''' - 8y = e^{ix}$

(b) $y^{(4)} + 16y = \cos x$.

18. If ϕ_1 is a solution of $L(y) = y'' + a_1(x)\,y' + a_2(x)\,y = 0$ on an interval I and $\phi_1(x) \neq 0$ on I, then show that a second solution of $L(y) = 0$ is

$$\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \frac{1}{[\phi_1(s)]^2} \exp\left[- \int_{x_0}^{s} a_1(t)\,dt \right] ds.$$

19. Derive the Bessel function of zero order of the first kind.

20. Let M and N be two real valued functions which have continuous partial derivatives on some rectangle.

$R : |x - x_0| \leq a,\ |y - y_0| \leq b$.

Prove that the equation $M(x, y) + N(x, y)\,y' = 0$ is exact in R, if and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

D–6874

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

First Semester

TOPOLOGY – I

(CBCS – 2018-19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Linear continuum.

2. Define the lower limit topology with an example.

3. What is meant by the subspace topology?

4. Define Order topology.

5. What is meant by project mapping?

6. Is the rationals Q connected? Justify.

7. Prove that any space X having only finitely many points is compact.

8. State the uniform limit theorem.

9. Define normal space.

10. Show that a subspace of a Lindelöf space need not be Lindelöf.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any three sets A, B, C, Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Or

(b) Prove that two equivalence classes E and E' are either disjoint or equal.

12. (a) If $\mathcal{B}$ is a basis for the topology on X and $\mathcal{C}$ is a basis for the topology of Y, then prove that $\mathcal{D} = \{B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

Or

(b) State and prove the Pasting lemma.

13. (a) Let $f : A \to X \times Y$ be defined by $f(a) = (f_1(a), f_2(a))$ prove that f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous.

Or

(b) State and prove sequencing lemma.

14. (a) State and prove Intermediate value theorem.

Or

(b) Prove that every compact subspace of a Hausdorff space is closed.

15. (a) Prove that every metrizable space is normal.

Or

(b) Define Hausdorff space. Prove that a product of Hausdorff space is Hausdorff.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that a countable union of a countable sets is countable.

17. Let X and Y be topological spaces ; let $f : X \to Y$. Prove that the following are equivalent :

    (a) f is continuous

    (b) for every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$.

    (c) For every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

18. Prove that the topologies on R^n induced by the Euclidean metric d and square metric ρ are the same as the product topology on R^n.

19. Prove that every regular space with a countable basis is normal.

20. State and prove Urysohn metrization theorem.

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D–6875

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics)DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Second Semester

ALGEBRA – II

(CBCS 2018–19 Academic Year onwards)

Time : Three hours | Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define vector space.

2. Define linear span.

3. Is any two finite dimensional vector space over F of the same dimension are isomorphic? Justify.

4. Define orthogonal complement of w.

5. Show that $w^{\perp}$ is a subspace of v.

6. Define algebraic extension of F.

7. Define trace of a matrix A in f_n.

8. Define unitary transformation.

9. Prove that the fixed field of G is a subfield of K.

10. Define Characteristics root of $T \in A(v)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) In a vector space show that $\alpha(v,w) = \alpha v - \alpha w$.

Or

(b) Prove that $L(s)$ is a subspace of v.

12. (a) If F is of Characteristics 0 and if a,b, are algebraic over F, then prove that there exists an element $c \in F(a,b)$ such that $F(a,b) = F(c)$.

Or

(b) If F is the field of real numbers, find $A(w)$ where w is spanned by (1,2,3) and (0,4,-1).

13. (a) State and prove Schwarz inequality.

Or

(b) Show that a polynomial of degree n over a field can have atmost n roots in any extension field.

14. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) If $u \in V_1$ is such that $uT^{n_1-k} = 0$, where $0 < k \le n$, then prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

15. (a) If $(v^T, v^T) = (v,v)$ for all $v \in V$ then prove that T is unitary.

Or

(b) If N is normal and AN = NA then prove that AN*=N*A.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions

16. If V and W are of dimensions m and n respectively, over F, then prove that Hon(V,W) is of dimension mn over F.

17. Let v be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

18. Prove that the number e is transcendental.

19. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

20. Prove that if V is n-dimensional over F and if $T \in A(v)$ has all its Characteristic roots in F, then T satisfies a polynomial of degree n over F.

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D–6876

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Second Semester

ANALYSIS – II

(CBCS 2018–19 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define upper and lower Riemann integrals of f over $[a,b]$.

2. Define unit step function.

3. Define convergent series of continuous functions having discontinuous sum with example.

4. Define algebra. Give an example.

5. Find M*A if A is countable.

6. Define orthonormal system.

7. Define null space and projection.

8. State Fatou's lemma.

9. If m*E=0 then prove that E is measurable.

10. Define Lebesgue integral.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) If f is continuous on $[a,b]$, then prove that $f \in \mathbb{R}(\alpha)$ on $[a,b]$

Or

(b) If f maps $[a,b]$ onto R^k and if $f \in \mathbb{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then prove that $\left|\int_a^b f dx\right| \leq \int_a^b |f| d\alpha$.

12. (a) If K is a compact metric space, if $f_n \in \zeta(k)$ for n=1,2,,... and if $\{f_n\}$ converges informly on k, then prove that $\{f_n\}$ is equicontinuous on K.

Or

(b) Prove that every uniformly converged sequence of bounded functions is uniformly bounded.

13. (a) State and prove localization theorem.

Or

(b) Prove that $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$.

14. (a) If $E = \bigcup_{n=1}^{\infty} E_n$, then prove that $\mu^*(E) \leq \sum_{n=1}^{\infty} \mu^*(E_n)$.

Or

(b) Let f and g be measurable real valued functions defined on x, let F be real and continuous on R^2, and take $h(x) = F(f(x), g(x))$, $x \in X$. Prove that h is measurable.

15. (a) State and prove monotone convergence theorem.

Or

(b) Prove that the continuous functions form a dense subset of $\mathcal{L}^2$ on $[a,b]$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that there exists a real continuous function on the real line which is nowhere differentiable.

17. State and prove stone Weierstrass theorem.

18. State and prove Parseval's

19. State and prove Little wood's third principle.

20. If f is a positive function on $(0,\infty)$ such that $f(x+1) = xf(x)$, $f(1) = 1$, logf is curved then prove that $f(x) = \sqrt{(x)}$.

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D–6877

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Second Semester

TOPOLOGY — II

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State countable intersection property.

2. Define compactification in a space.

3. Define stone cech compactification.

4. Let $A \subset X$, let $f : X \to Z$ be a continuous map of A into the Hausdorff space Z. Prove that there is atmost one extension of f to a continuous function $g : \overline{A} \to Z$.

5. Prove that the collection $A = \{(n, n+2) / n \in Z\}$ is locally finite.

6. Is the space Q of rational numbers with metric $d(x, y) = |x - y|$, complete? Justify.

7. Define uniform metric on y^J.

8. When will you say a space X is topologically complete?

9. Define point open topology.

10. State Ascoli's theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a set ; let D be a collection of subsets of X that is maximal with respect to the finite intersection property. Show that $x \in \overline{D}$ for every $D \in D$ if and only if every neighborhood of x belongs to D.

Or

(b) Define locally compact space with an example. Show that Q of rationals are not locally compact.

12. (a) Under what conditions does a metrizable space have a metrizable compactification?

Or

(b) Let X be completely regular. Prove that X is connected if and only if $\beta(x)$ is connected.

13. (a) Prove that a product of completely regular space is completely regular.

Or

(b) Prove that every paracompact Hausdorff space X is normal.

14. (a) Prove that the Euclidean space R^k is complete in either of its usual metrics, the Euclidean metric d or the square metric ρ.

Or

(b) Let X be a space and let (y, d) be a metric space. If the subset f of $\zeta\,(x, y)$ is totally bounded under the uniform metric corresponding to d, then prove that f is equicontinuous.

15. (a) Show that in the compact open topology, $\zeta\,(x, y)$ is Hausdorff if y is Hausdorff.

Or

(b) Let $(_1)(_2)$.... be a nested sequence of non-empty closed sets in the complete metrics space X. If $\dim c_n \to 0$ then prove that $nc_n \neq \varphi$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tychonoff theorem.

17. Let X be a metrizable space. If $\mathcal{A}$ is an open covering of X, then prove that there is an open covering ξ of X refining $\mathcal{A}$ that is countably locally finite.

18. Prove that a space X is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable.

19. State and prove Ascoli's theorem.

20. State and prove Baire category theorem.

D–6878

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018–19 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz), cz - ax, ax - by)$.

2. Solve $yz\,dx + xz\,dy + xy\,dz = 0$.

3. Eliminate the arbitrary constants a and b from $z = (x + a)(y + b)$.

4. Form the partial differential equation by eliminating the arbitrary function from $z = x + y + f(xy)$.

5. Find a particular integral of $(D^2 - D^1)z = e^{2x+y}$.

6. Show that $\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 0$ is parabolic.

7. State the interior Neumann problem.

8. Find a complete integral of $pq = 1$.

9. Show that the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ is integrable.

10. When we say that the equation $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ is elliptic.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Find the integral curves of the sets of equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}.$$

Or

(b) Solve: $(y+z)dx + (z+x)dy + (x+y)dz = 0$.

12. (a) Form the partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = 1$.

Or

(b) Find the general integral of $z(xp - yq) = y^2 - x^2$.

13. (a) Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them.

Or

(b) Find a complete integral of $p^2x + q^2y = z$, by Jacobi's method.

14. (a) Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}.$$

Or

(b) Find the D'Alembert's solution of the one dimensional wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.

15. (a) Find the solution of one dimensional diffusion equation $\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{k}\frac{\partial \theta}{\partial t}$.

Or

(b) Prove that if ψ is continuous within and on the circumference of a circle and is harmonic in the interior, then the value of ψ at the centre is equal to the mean value on the boundary.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

16. Verify that the differential equation

$(y^2 + yz)\,dx + (xz + z^2)\,dy + (y^2 - xy)\,dz = 0$ is integrable and find its primitive.

17. From

(a) $x^2 + y^2 + (z-c)^2 = a^2$,

(b) $x^2 + y^2 = (z-c)^2 \tan^2 \alpha$ and

(c) $z = f(x^2 + y^2)$, show that they have same partial differential equation.

18. Find the integral surface of the equation $px + qy = z$ passing through $x + y = 1$ and $x^2 + y^2 + z^2 = 4$.

19. A uniform string of line density ρ stretched to tension $\rho\, c^2$ and executes a small transverse vibration in a place through the undisturbed line of the string. The ends $x = 0$ and $x = l$ of the string are fixed. The string is at rest, with the point $x = b$ drawn aside through a small distance E and released at time $t = 0$. Find the transverse displacement y at any time t by Fourier series method.

20. The faces $x = 0$ and $x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $\theta = f(x)$, $0 \le x \le a$. Determine the temperature at a subsequent time t.

D–6879

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define torsion of a curve.
2. Define evolute of a space curve.
3. Define osculating sphere.
4. What is meant by right helicoid?
5. Define the tangential components.
6. State the fundamental existence theorem for space curves.
7. Define the geodesic curvature.

8. When will you say that a vector is called the geodesic vector of the curve?

9. Define the characteristic line.

10. Define osculating developable of the curve.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If the radius of spherical curvature is constant, then prove that the curve either lies on a sphere or has constant curvature.

Or

(b) With the usual notations, prove that $[\vec{v}', \vec{v}'', \vec{v}'''] = k^2 \tau$

12. (a) Show that the involutes of a circular helix are plane curves.

Or

(b) Find the area of the anchor ring.

13. (a) Show that a curve on a surface is geodesic if and only if its Gaussian curvature vector is zero.

Or

(b) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z = constant.

14. (a) Derive the Liouville's formula for Kg.

Or

(b) Prove that every helix on a cylinder is a geodesic.

15. (a) Discuss the Dupin's indicatrix.

Or

(b) Enumerate the polar developable.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Discuss an isometric correspondence in detail.

17. Find the intrinsic equations of the curve given by $x = a\ e^{u} \cos\ u, y = a\ e^{u} \sin\ u, z = b\ e^{u}$

18. If θ is the angle at the point (u,v) between the two directions given by $P\ du^2 + 2Q\ dudv + R\ dv^2 = 0$, then prove that $\tan\theta = \dfrac{2H\sqrt{Q^2 - PR}}{ER - 2FQ + GP}$.

19. Derive the Christoffel symbols of the second kind.

20. Derive the Rodrigue's formula.

D–6880

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

Mathematics

OPTIMIZATION TECHNIQUES

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Write down the types of algorithm in Network models.
2. Define a critical activity.
3. Define the effective lead time.
4. Write the formula for purchasing cost per unit time in EOQ with price breaks.
5. Define a queue discipline.
6. Write down the P-K formula.
7. Draw the transition - rate diagram.
8. Define strategies of a game and value of the game.
9. When we say that a function $f(x_1, x_2, ..., x_n)$ is separable.
10. What is quadratic programming?

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Describe Dijkstra's algorithm.

Or

(b) Construct the network diagram comprising activities B, C, ...Q and N such that the following constraints are satisfied. B<E, F; C<G,L; E,G<H; L,H<I; L<M; H<N; H<J; I,J,<P; P<Q. The notation X<Y means that the activity x must be finished before y can begin.

12. (a) For what value of λ, the game with the following payoff matrix is strictly determinable?

Player B

Player A		B_1	B_2	B_3
	A_1	λ	6	2
	A_2	-1	λ	-7
	A_3	-2	4	λ

Or

(b) Solve the following game and determine the value of the game

$$\begin{array}{c} \quad B \\ A\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \end{array}$$

13. (a) Solve the following L.P.P:

Maximize $z = 3x_1 + 9x_2$

Subject to the constraints:

$x_1 + 4x_2 \le 8,$

$x_1 + 2x_2 \le 4;$

$x_1, x_2 \ge 0$

Or

(b) Explain the bounded variable algorithm.

14. (a) Show, how the following problem can be made separable.

Maximize: $z = x_1, x_2, + x_3 + x_1 x_3$

Subject $x_1, x_2 + x_3 + x_1 x_3 \le 0$

$x_1, x_2, x_3 \ge 0$.

Or

(b) Explain the solution of mixed strategy.

15. (a) Solve the non-linear programming problem:

Minimize: $f(x_1, x_2) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$

Subject to the constraints: $2x_1 - x_2 = 4$

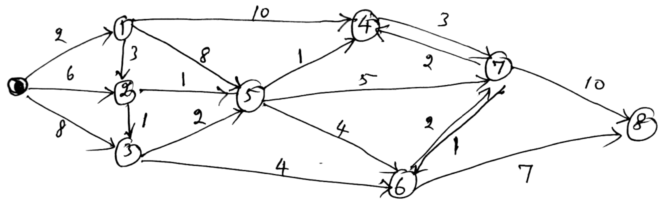
Or

(b) Discuss briefly about "separable convex programming".

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Find the shortest route from 0 to 8. The numbers on the links represent the distance in kilometres.



17. Using the bounded variable technique, solve the following L.P.P.

Maximize : $z = 3x_1 + x_2 + x_3 + 7x_4$

Subject to the constraints

$2x_1 + 3x_2 - x_3 + 4x_4 \le 40$

$-2x_1 + 2x_2 + 5x_3 - x_4 \le 35$

$x_1 + x_2 - 2x_3 + 3x_4 \le 100;$

$x_, \ge 2,\ x_2 \le 1, x_3 \ge 3, x_4 \ge 4.$

18. Solve by revised simplex method:

minimize: $z = x_1 + 2x_2$

Subject to the constraints:

$2x_1 + 5x_2 \geq 6,$

$x_1 + x_2 \geq 2;$

$x_1 \geq 1$ and $x_2 \geq 0$

19. Use the method of Lagragian multipliers to solve the non-linear programming problem:

Maximize $z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to the constraints: $x_1 + x_2 + x_3 = 20$

20. Solve the quadratic programming problem:

Maximize: $z = 2x_1 + 3x_2 = 2x_1^2$

Subject to the constraints:

$x_1 + 4x_2 \leq 4.$

$x_1 + x_2 \leq 2;$

$x_1, x_2 \geq 0$

D–6881

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. If a prime p does not divide a, then show that $(p, a) = 1$.

2. Find all integers n such that $\varphi(n) = 12$.

3. State Mobius inversion formula.

4. Define Liouville's function $\lambda(n)$ and the divisor function $\sigma_{\alpha}(n)$.

5. If f is multiplicative then show that; $f(1) = 1$.

6. Prove that $\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F$.

7. Define average order of $\mu(n)$ and of $\wedge(n)$.

8. If $a \equiv b(\mathrm{mod}\, m)$ and if $0 \leq |b-a| < m$, then show that $a = b$.

9. If p is odd, $p > 1$, prove that $1^2.3^2.5^2...(p-2)^2 \equiv (-1)^{\frac{p+1}{2}} (\mathrm{mod}\, p)$

10. State Wolskenholme's theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) State and prove Euclid's lemma.

Or

(b) Prove that if $n \geq 1$, $\sum_{d/n} \mu(d) = \begin{cases} 1 & 1\ if\ \ n = 1 \\ 0 & if\ \ n > 1 \end{cases}$.

12. (a) Prove that $n \geq 1$ $\varphi(n) = \sum_{d/n} \mu(d)(n/d)$.

Or

(b) State and prove division algorithm.

13. (a) If f and g are multiplicative, then prove that its Dirichlet product is $f * g$.

Or

(b) State and prove generalized Mabius inversion formula.

14. (a) Assume $(a,m) = d$. Prove that the linear congruence $ax \equiv b(\mathrm{mod}\, m)$ has solutions if, and only if d/b.

Or

(b) State and prove Euler's summation formula.

15. (a) Lets p be an off prime. Then prove that for all n, $(n/p) = n^{(p-1)/2} (\text{mod } p)$.

Or

(b) State and prove Wilson's theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Fundamental theorem of arithmetic.

17. (a) Prove that if $n \geq 1$, $\varphi(n) = n \underset{p/n}{\pi} (1 - \frac{1}{p})$.

(b) Prove that , if $2^n - 1$ is prime, then n is prime.

18. If both g and $f * g$ are multiplicative, then prove that f is also multiplicative.

19. Prove that for all $x \geq 1$, $\sum_{n \leq x} d(n) = x \log x + (2c - 1)x + 0(x)$ where C is Euler's constant.

20. State and prove Lagrange' theorem.

D–6882

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define order of a Markov chain.

2. Define non-full persistent state.

3. Define transition densities.

4. State the Brownian motion.

5. Write the forward diffusion equation.

6. Write the equation of motion of a Brownian particle.

7. Define traffic intensity.

8. Define probability of extinction.

9. Define idle period.

10. Write the Erlang's second formula.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) The t.p.m of a Markov chain $\{X_n, n = 1,2,\ldots\}$ having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $\prod_0 = (0.7, 0.2, 0.1)$. Find

(i) $\Pr(X_2 = 3)$

(ii) $\Pr\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

Or

(b) If $\{N(t)\}$ is a Poisson process then prove that the autocorrelation coefficient between $N(t)$ and $N(t+s)$ is $\left(\frac{t}{t+s}\right)^{1/2}$.

12. (a) If $X(t)$ with $X(0)$ and $\mu = 0$ is a Wiener process and $0 < s < t$, show that for atleast one λ satisfying $s \le \lambda \le t$ $\Pr\{X(\tau) = 0\} = \left(2/\pi\right)\cos^{-1}\left(\left(\frac{s}{t}\right)^{1/2}\right)$.

Or

(b) Let $\{X(t), t \ge 0\}$ be a Wiener process with $\mu = 0$ and $X(0) = 0$. Find the distribution of T_{a+b} for $0 < a < a + b$.

13. (a) Prove that the p.g.f $R_n(s)$ of Y_n satisfies the recurrence relation $R_n(s) = s\,P(R_{n-1}(s))$; $P(s)$ being the p.g.f of the offspring distribution.

Or

(b) Show that the p.g.f. of the conditional distribution of X_n, given $X_n > 0$, is $\dfrac{P_n(s) - P_n(0)}{1 - P_n(0)}$.

14. (a) Prove that $E\{X_{n+r} / X_n\} = X_n m^r$ for $r, n = 0,1,2,\ldots$.

Or

(b) Derive the state solution.

15. (a) Derive the necessary and sufficient condition for the existence of a steady state of the infinite series $\sum_{n=1}^{\infty} \prod_{K=1}^{n} \dfrac{\lambda_{k-1}}{\mu k}$.

Or

(b) Show that the average number of busy channels in the system for $M / M / \infty$ model is λ/μ.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the Chapman – Kolmogorov equation.

17. Let a finite Markov chain with state space $S = \{0,1,2,\ldots,l\}$ be a martingale. Prove that, as $n \to \infty$,

$$P_{ij}^{(n)} = 0, \qquad j = 1,2,\ldots,l-1 \qquad \text{and} \qquad \left.\begin{aligned} P_{il}^{(n)} &= \frac{i}{l} \\ P_{io}^{(n)} &= 1 - \frac{i}{l} \end{aligned}\right\},$$

$i = 1,2,\ldots,l-1$.

18. Show that Ornstein – Uhlenbech process as a transformation of Wiener process.

19. Suppose that $m = 1$ and $\sigma^2 < \infty$, then prove that

(a) $\lim_{n\to\infty} n \cdot \Pr\{X_n > 0\} = \frac{2}{\sigma^2}$

(b) $\lim_{n\to\infty} E\left\{\frac{X_n}{n} / X_n > 0\right\} = \frac{\sigma^2}{2}$ and

(c) $\lim_{n\to\infty} \Pr\left\{\frac{X_n}{n} > u / X_n > 0\right\} = \exp\left\{-\frac{2u}{\sigma^2}\right\}, \ u \geq 0$.

20. A mechanic looks after 8 automatic machines, a machine breaks down, independently of others, in accordance with a Poisson process, the average length of time for which a machine remains in working order being 12 hours. The duration of time required for repair of a machine has an exponential distribution with mean 1 hour.

Find

(a) The probability that 3 or more machines will remain out of order at the same time.

(b) The average number of machines in working order and

(c) For what fraction of time, on the average, the mechanic will be idle?

D–7328

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define a complete bipartite graph. Give an example.

2. Prove that in a tree any two vertices are connected by a unique path.

3. Define disconnected graph with an example.

4. Find the number of different perfect matchings in K_{2n}.

5. Define edge chromatic number.

6. When will you say a graph is critical graph?

7. Define $\gamma(k,l)$.

8. Prove that K_5 is non-planar.

9. Define indegree and outdegree of a vector v in a digraph.

10. Define a Hamiltonian path in a digraph.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If a k-regular bipartite graph with $k > 0$ has bipartition (X, Y), then prove that $|X| = |Y|$.

Or

(b) If G is a tree, prove that $\varepsilon = \gamma - 1$.

12. (a) Prove that $\gamma(m, n) = \gamma(n, m)$.

Or

(b) Let G be a k-regular bipartite graph with $k > 0$. Prove that G has a perfect matching.

13. (a) Find the edge chromatic number of K_n and $K_{m,n}$.

Or

(b) If G is uniquely n-colourable, then prove that $\delta(G) \geq n - 1$.

14. (a) Show that there is no map of five regions in the plane such that every pair of regions are adjacent.

Or

(b) Explain the four colour conjecture.

15. (a) Prove that the $(i, j)^{th}$ entry A^n is the number of walks length n from v_i and v_j.

Or

(b) Prove that in a simple digraph, every vertex lies in exactly one strong component.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that a vertex V of a tree G is a cut vertex of G if and only if $\deg(v) > 1$.

17. Prove that a connected graph has an Euler trial if and only if it has atmost two vertices of odd degree.

18. State and prove Vizing's theorem.

19. Prove that every planar graph is 5-vertex colourable.

20. State and prove max-flow, min-cut theorem.

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D–7329

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Banach space. Give an example.

2. Define quotient norm on X/Y.

3. Give an example of a discontinuous linear functional.

4. Define Hamel basis.

5. Define inner product space. Give an example.

6. Define annihilator.

7. Define unitary operator.

8. Define orthogonal projection.

9. Write Schwartz inequality.

10. State the open mapping theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Show that the real linear space $\mathbb{R}$ and the complex linear space $\mathbb{C}$ are Banach spaces under the norm $\|x\| = |x|$, $x \in \mathbb{R}$ or $\mathbb{C}$

Or

(b) Let x be a normal space, and let the subset $\{x \in X / \|x\| \leq 1\}$ be compact in x. prove that X is finite dimensional.

12. (a) Prove that if x is a finite dimensional linear space then all linear functionals are bounded.

Or

(b) Let X and Y be normed spaces. Prove that if X is finite dimensional then every linear map from X to Y is continuous.

13. (a) Let $< \cdot >$ be an inner product on a linear space X. Prove that $4 < x, y > = < x + y, x + y > - < x - y, x - y > + i < x + iy, x + iy > - i < x - iy, x - iy >$ for all $x, y \in X$.

Or

(b) Prove that every completely continuous linear transformation is a continuous linear transformation.

14. (a) If f is a linear functional on the Hilbert space X, with null space N, then prove that f is continuous if and only if N is a closed subspace.

Or

(b) Prove that $\|A^*A\| = \|A\|^2$, A is a bounded linear transformation.

15. (a) State and prove the closed graph theorem.

Or

(b) Let A be a symmetric operator. Support that $R(A) = x$ that is; A is an onto mapping. prove that A is self-adjoint.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Let X be a normed linear space. then show that the following conditions are equivalent

(a) Every closed and bounded subset of X is compact.

(b) The subset $\{x \in X / \|x\| \leq 1\}$ of X is compact

(c) X is finite dimensional.

17. Let $\widetilde{X}$ denote the normed linear space of all bounded linear functionals over the normed linear space X. Prove that $\widetilde{X}$ is a Banach space.

18. Let $\{u_1, u_2, ...\}$ be a countable orthonormal set in an inner product space X and $x \in X$. Prove that $\sum_n |\langle x, u_n \rangle|^2 \leq \|x\|^2$, where equality holds if and only if $x = \sum_n \langle x, u_n \rangle u_n$.

19. State and prove Hahn-Banach theorem.

20. State and prove open mapping theorem.

D–7330

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Jacobian matrix.
2. Define a linear system with an example.
3. Define Eigen function.
4. Define the matrix norm.
5. State the interpolating conditions.
6. Define the shape function $N_i(x)$.
7. Write the Hermite interpolating polynomial.
8. State Weierstrass approximation theorem.
9. Write down the order of the single step method.
10. State the boundary value problem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the multiple root of the equation $f(x) = 27x^5 + 27x^4 + 36x^3 + 28x^2 + 9x + 1 = 0$ by Newton-Raphson method.

Or

(b) Use synthetic division and perform two iteration by Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$.

12. (a) Solve the following systems of equations buy Gauss Elimination method.

$$4x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 - 2x_3 = 4$$
$$3x_1 + 2x_2 - 4x_3 = 6$$

Or

(b) Estimate the Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$, using the Gerschgorin bounds.

13. (a) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 1 & 2 \\ -1 & 2 & -1 \end{bmatrix}$ by Cholesky method.

Or

(b) Using piecewise linear interpolation, find the interpolating polynomial for the following data:

x	0	1	2
y=f(x)	1	3	35

14. (a) Determine the best minimax approximation to $e^{|x|}$ with a polynomial of degree 0 and 1 for $|x| \le 1$

Or

(b) Evaluate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ using Gauss-Legendre three point formula.

15. (a) Use the Euler method to solve numerically the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0,1]$.

Or

(b) Apply the Taylor's series second order method to integrate $y' = 2t + 3y$, $y(0) = 1$, $t \in (0, 0.4)$ with $h = 0.1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all the roots of the polynomial $x^3 - 4x^2 + 5x - 2 = 0$ using the Graeffe's root squaring method.

17. Find the largest Eigen value in modulus and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$, using the power method.

18. Obtain the cubic Spline approximation for the function given in the tabular form

x	0	1	2	3
f(x)	1	2	33	244

19. Calculate $\int_0^{0.8} (1 + \frac{\sin x}{x}) dx$, correct to 5 decimal places.

20. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0,1]$, by using the second order implicit Runge-Kutta method.

D–7331

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. For each $c \subset \mathcal{C}$, prove that $p(c) - 1 - p(c^*)$

2. Define conditional probability.

3. Let $f(x, y) = \begin{cases} 6x^2 y, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

   Find $\Pr\left(0 < x < \frac{3}{4}, \frac{1}{3} < y < 2\right)$

4. Prove that the expected value of the product of two random variables is equal to the product of their expectations plus their covariance.

5. Write the p.d.f of Gamma distribution.

6. Define covariance of X and Y.

7. If the p.d.f of X is $f(x) = \begin{cases} 2x\,e^{-x^2}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$, then find the p.d.f. of $Y = X^2$.

8. Write down the m.g.f. of $Y = \sum_{i=1}^{n} X_i$.

9. Define t – distribution.

10. Define convergence in probability distribution.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the Baye's theorem.

Or

(b) Find the mean and variance of the p.d.f.

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

12. (a) Let the joint p.d.f of X_1 and X_2 be

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that the random variables X_1 and X_2 are dependent.

Or

(b) Let $f(x_1, x_2) = \begin{cases} 4x_1\,x_2, & 0 < x_1 < 1;\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$

be the p.d.f. of X_1 and X_2. Find $\Pr\left(0 < X_1 < \frac{1}{2},\ \frac{1}{4} < X_2 < 1\right)$.

13. (a) Find the measures of skewness and kurtosis of the Binomial distribution $b(n,\ p)$.

Or

(b) Let X have a gamma distribution with $\alpha = \frac{\gamma}{2}$ where γ is a positive integer, and $\beta > 0$. Define the random variable $Y = 2X/\beta$. Show that the p.d.f. of Y is $\chi^2(\gamma)$.

14. (a) Derive the p.d.f. of Chi-square distribution.

Or

(b) Let X and Y be random variables with $\mu_1 = 1$, $\mu_2 = 4$, $\sigma_1^2 = 4$, $\sigma_2^2 = 6$, $\rho = \frac{1}{2}$ Find the mean and variance of $Z = 3X - 2Y$.

15. (a) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

Or

(b) Let Z_n be $\chi^2(n)$ and let $W_n = \frac{Z_n}{n^2}$, find the limiting distribution of W_n.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Chebyshev's inequality theorem.

17. Let $f(x, y) = \begin{cases} e^{-x-y}, & 0 \le x < \infty,\ 0 \le y < \infty \\ 0, & \text{elsewhere} \end{cases}$

be the joint p.d.f of X and Y.

Find $P[X < 1]$, $P[X > Y]$ and $P[X + Y = 1]$

18. Find the m.g.f. of a normal distribution and hence find the mean and variance of a normal distribution.

19. Derive the p.d.f. of F-distribution.

20. State and prove the central limit theorem.

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D–5511

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

ALGEBRA – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the Euclidean algorithm.

2. Define automorphism of a group.

3. Show that the group of order 21 is not simple.

4. If $o(G) = p^2$, where p is a prime, show that G is abelian.

5. Define an integral domain.

6. If U is an ideal of R and $1 \in U$, then prove that $U = R$.

7. Define a left ideal of R.

8. When will you say a polynomial is integermonic?

9. Prove that any field is an integral domain.

10. Find the g.c.d of the polynomial $x^2 + 1$ and $x^6 + x^3 + x + 1$ in $Q[x]$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ for any three sets A, B and C.

Or

(b) Let $\theta : G \to H$ be a onto group homomorphism with Kernal K. Prove that G/K is isomorphic to H.

12. (a) State and prove the Cauchy's theorem for abelian group.

Or

(b) If P is a prime number and $P / o(G)$, then prove that G has an element of order P.

13. (a) Show that any two p-Sylow subgroups of a group G are conjugate.

Or

(b) If D is an integral domain and D is of finite characteristic, prove that the characteristic of D is a prime number.

14. (a) Show that the set of multiples of a fixed prime number p form a maximal ideal of the ring of integers.

Or

(b) If $[a,b] = [a',b']$ and $[c,d] = [c',d']$, then prove that $[a,b][c,d] = [a',b'][c',d']$.

15. (a) Prove that $J[i]$ is a Euclidean ring.

Or

(b) State and prove the Einstein criterian theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If G is a group, N a normal subgroup of G, then prove that G/N is also a group.

17. Prove that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S.

18. If Q is a homomorphism of R into R' with Kernal $I(Q)$ then prove the following

(a) $I(Q)$ is a subgroup of R under addition

(b) If $a \in I(Q)$ and $\gamma \in R$, then both $a\gamma$ and γa are in $I(Q)$.

19. If U is an ideal of a ring R, then prove that R/U is a ring and is a homomorphic image of R.

20. If R is an integral domain, then prove that $R[x_1, x_2, \ldots, x_n]$ is an integral domain.

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D–5512

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define countable and uncountable set.

2. For $x \in R'$ and $y \in R'$, define $d_1(x,y) = (x-y)^2$, determine, whether it is a metric or not.

3. Define compact set. Give an example.

4. When do you say that a series converge? Give an example of a divergent series.

5. State the root test.

6. Define bounded and unbounded function.

7. Give an example of a function which has second kind discontinuity at every point.

8. Find the radius of convergence of the power series $\sum \frac{2^n}{n^2} z^n$.

9. State the generalized mean value theorem.

10. Let f be defined on $[a,b]$. If f is differentiable at a point $x \in [a,b]$, then prove that f is continuous at x.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $|z+w| \le |z| + |w|$, where z and w are complex numbers.

Or

(b) Prove that a set E is open if and only if its complement is closed.

12. (a) Define a Cauchy sequence. If X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X, then prove that $\{p_n\}$ converges to some point of X.

Or

(b) State and prove the Weierstrass theorem.

13. (a) Derive the partial summation formula.

Or

(b) Suppose (i) the partial sums A_n of $\sum a_n$ form a bounded sequence (ii) $b_0 \ge b_1 \ge \ldots \ge b_n \ge \ldots$, (iii) $\lim_{n\to\infty} b_n = 0$. Prove that $\sum a_n \ b_n$ converges.

14. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

Or

(b) Suppose f is a continuous mapping of metric space X into a metric space Y. If E is a connected subset of X, then prove that $f(E)$ is connected.

15. (a) State and prove chain rule for differentiation.

Or

(b) State and prove Cauchy mean-value theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that for every real $x > 0$ and every integer $n > 0$, there is one and only one real y such that $y^n = x$.

17. Prove that every k-cell is compact.

18. Investigate the behaviour (convergence or divergence) of $\sum a_n$ if

(a) $a_n = \dfrac{\sqrt{n+1} - \sqrt{n}}{n}$

(b) $a_n = \left(\sqrt[n]{n} - 1\right)^n$.

19. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

20. State and prove the inverse function theorem.

D–5513

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State the existence theorem for linear equations with constant coefficients.

2. Solve $y'' - 4y = 0$.

3. Compute the Wronskian of $\varphi_1(x) = x^2$, $\varphi_2(x) = 5x^2$.

4. Define indicial polynomial.

5. Show that $P_n(-x) = (-1)^n P_n(x)$.

6. Define singular point.

7. Find all real valued solution of the equation $y' = x^2 y$.

8. Show that $f(x, y) = y^{1/2}$ does not satisfy the Lipschitz condition on $R : |x| \leq 1,\ 0 \leq y \leq 1$.

9. Consider the initial value problem $y' = y_2^{(2)} + 1$, $y_2' = y_1^{(2)}$; $y_1(0) = 0$, $y_2(0) = 0$, Compute the first three successive approximations $\varphi_0, \varphi_1, \varphi_2$.

10. State the non-local existence theorem for $y' = f(x, y)$, $y(x_0) = y_0$, $(|y_0| < \infty)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve : $y'' + 10y = 0$, $y(0) = \pi$, $y'(0) = \pi^2$.

Or

(b) Find all solutions of $y'' - 4y' + 5y = 3e^{-x} + 2x^2$.

12. (a) Show that $\int_{-1}^{1} P_n^2(x)\,dx = \dfrac{2}{2n+1}$.

Or

(b) Let φ_1, φ_2 be two solutions of $L(y) = 0$ on an interval I and let x_0 be any point in I. Prove that φ_1, φ_2 are linearly independent on I if and only if $W(\varphi_1, \varphi_2) \neq 0$.

13. (a) Find all solutions of the equation $x^2 y'' + xy' - 4\pi y = x$ for $x > 0$.

Or

(b) Show that $\int_{-1}^{1} P_n(x) P_m(x)\,dx = 0$ when $n \neq m$.

14. (a) Show that –1 and +1 are regular singular points for the Legendre equation $(1-x^2)y''-2xy'+\alpha(\alpha+1)y=0$.

Or

(b) Compute the first four successive approximation $\varphi_0, \varphi_1, \varphi_2, \varphi_3$ for the equation $y'=1+xy,\ y(0)=1$.

15. (a) Let $f(x, y)=\frac{\cos y}{1-x^2}$, $(|x|<1)$. Show that f satisfies a Lipschitz condition on every strip $S_\alpha : |x| \le \alpha$, where $0<\alpha<1$.

Or

(b) Prove that a function φ is a solution of the initial value problem $y'=f(x,y),\ \ y(x_0)=y_0$ on an interval I if and only if it is a solution of the integral equation $y=y_0+\int_{x_0}^{x} f(t, y)\,dt$ on z.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let φ be a solution of $L(y)=y^{(n)}+a_1y^{(n-1)}+....+a_ny=0$ on an interval I containing a point x_0. For all x in I, prove that $\|\varphi(x_0)\|e^{-k|x-x_0|} \le \|\varphi(x)\| \le \|\varphi(x_0)\|e^{k|x-x_0|}$, where $k=1+|a_1|+|a_2|+....+|a_n|$.

17. Find the two linearly independent power series solutions of the equation $y''-xy'+y=0$.

18. Derive Bessel's function of first kind of order $\alpha_i\ J_\alpha(x)$.

19. Find all solutions of the equation

$x^2 y'' - (2+i)xy' + 3iy = 0$ for $x > 0$

20. Let f be a real-valued continuous function on the strip $S : |x - x_0| \le a$, $|y| < \infty$, $a > 0$ and f satisfies on S a Lipschitz condition with constant $k > 0$. Prove that the successive approximations $\{\varphi_k\}$ for the problem $y' = f(x, y)$, $y(x_0) = y_0$ exist on the entire interval $|x - x_0| \le a$ and converge there to a solution φ of $y' = f(x, y)$, $y(x_0) = y_0$.

D–5514

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define one-one function, Give an example.

2. Define equivalence class.

3. Define well-ordered set.

4. What is meant by quotient map?

5. Is the rational Q connected? Justify your answer.

6. Define compact space. Give an example.

7. What is meant by the path component of the space X?

8. Define limit point compact.

9. Define regular space.

10. Whether the space $\mathbb{R}_k$ is Hausdorff or not? Justify your answer.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a countable union of countable set is countable.

Or

(b) State and prove strong induction principle.

12. (a) Let A be a subset of a topological space X and let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

Or

(b) State and prove the uniform limit theorem.

13. (a) Let X be locally path connected. Prove that every connected open set X is path connected.

Or

(b) State and prove the sequence lemma.

14. (a) Let Y be a subspace of X. Prove that Y is compact if and only if every covering of Y by sets open in X contains a finite subcollection covering Y.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Prove that every locally compact Hausdorff space is completely regular.

Or

(b) Define regular space. Prove that a subspace of a regular space is regular.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Define the Cartesian product of an indexed family of sets $\{A_\alpha\}_{\alpha\in J}$. Prove that a finite product of countable set is countable.

17. Prove that every finite point set in a Hausdorff space is closed.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L.

19. State and prove tube lemma.

20. State and prove Urysohn metrization theorem.

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D–5515

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define dual space.

2. Define the orthogonal complement.

3. Find the degree of the splitting field of $x^4 - 2$ over F.

4. Express the polynomial $x_1^3 + x_2^3 + x_3^3$ in the elementary symmetric functions in x_1, x_2, x_3.

5. When will you say that an element in $A(V)$ is singular?

6. Define Hermition and skew hermition.

7. What is meant by index of nilpotence?

8. Define the elementary divisors of T in $A(V)$.

9. Define symmetric and skew symmetric matrix.

10. Define unitary transformation.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $L(S)$ is a subspace of a vector space V.

Or

(b) With usual notations, prove that $F^{(n)}$ is isomorphic with $F^{(m)}$ if and only if $n = m$.

12. (a) State and prove the remainder theorem.

Or

(b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

13. (a) For any $f(x), g(x) \in F[x]$, prove that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

Or

(b) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

14. (a) Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $V \neq 0$ in V, $vT \equiv \lambda v$.

Or

(b) Let V be finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of minimal polynomial for T is not 0.

15. (a) Prove that every $A \in F_n$ satisfies its characteristic equation.

Or

(b) Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If V and W are of dimensions m and n respectively, over F, then prove that $Hom(V,W)$ is of dimension mn over F.

17. Let V be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

18. Prove that the number e is transcendental.

19. If $p(x) \in F[x]$ is solvable by radicals over F, then prove that the Golois group over F of $p(x)$ is a solvable group.

20. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

D–5516

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

ANALYSIS – II

(CBCS 2018-19 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Prove that any constant function is Riemann integrable.

2. Define an equicontinuous function on a set.

3. Show that $(e^x)^1 = e^x$.

4. Define pointwise bounded function.

5. On what intervals does the series $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2x}$ converge uniformly?

6. Define an orthogonal system of functions on $[a, b]$.

7. Define the gamma function.

8. Define Borel set.

9. Define measurable function. Give an example.

10. Define an integrable function over the measurable set.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if f is continuous on $[a,b]$ then $f \in R(\alpha)$. on $[a,b]$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) State and prove the Cauchy criterion for uniform convergence.

Or

(b) Let β be the uniform closure of an algebra A of bounded functions. Prove that B is a uniformly closed algebra.

13. (a) If f is a positive function on $(0, \infty)$ such that

(i) $f(x+1) = x(f(x))$

(ii) $f(1) = 1$

(iii) $\log f$ is convex, then show that $f(x) = \sqrt{x}$.

Or

(b) If f is a continuous and if $\in > 0$, then prove that there is a trigonometric polynomial P such that $|p(x) - f(x)| < \in$ for all real x.

14. (a) State and prove Egoroff's theorem.

Or

(b) Let $< E_n >$ be an infinite decreasing sequence of measurable sets. Let ${}_m E_1$ be finite, then prove that $m\left(\overset{\infty}{\underset{i=1}{\wedge}} \in i\right) = \lim_{n \to \infty} {}_m E_n$.

15. (a) Let φ and ψ be simple functions which vanish outside a set of finite measure. Prove that $\int(a\varphi+b\psi)=a\int\varphi+b\int\psi$ and if $\varphi\geq\psi$ almost everywhere then prove that $\int\varphi\geq\int\psi$.

Or

(b) State and prove Fatou's lemma.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that there exists a real continuous function on the real line which is no where differentiable..

17. State and prove Parseval's theorem.

18. Prove that the outer measure of an interval is its length.

19. Let f and g be integrable over E. Prove the following:

(a) The function *cf* is integrable over E and $\int_E cf = c\int_E f$.

(b) The function $f+g$ is integrable over E and $\int_E f+g = \int_E f + \int_E g$.

(c) If $f \leq g$ almost everywhere, then $\int_E f \leq \int_E g$.

20. State and prove Lebesque monotone convergence theorem.

D–5517

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

TOPOLOGY – II

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a completely regular space.

2. When will you say two compactification is equivalent?

3. Define locally finite in a topological space. Give an example.

4. Define paracompact space.

5. What is meant by point open topology?

6. Define an equicontinuous function.

7. Define compactly generated space.

8. What is meant by compact open topology?

9. Show that the set Q of rationals is not a G_8-set in the reals.

10. Define a finite dimensional space.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the subspace of a completely regular space is completely regular.

Or

(b) If X is completely regular then prove that x can be imbedded in $[0, 1]^J$ for some J.

12. (a) Prove that a metric space x is complete if every Cauchy sequence in x has a convergent subsequence.

Or

(b) Show that the metric space (x, d) is complete if and only if for any nested sequence $A_1 \supset A_2 \supset \ldots$ of non empty closed sets of X such that diameter $A_n \to 0, \underset{n \in z_+}{n} A_n \neq \varphi$.

13. (a) Prove that every metrizable space is paracompact.

Or

(b) Let X be a compactly generated space; let (y, d) be a metric space, prove that $\zeta(x, y)$ is closed in y^x in the topology compact convergence.

14. (a) If X is a compact Hausdorff space, or a complete metric space, then prove that x is a Baire space.

Or

(b) Show that every locally compact Hausdorff space in a Baire space.

15. (a) Show that the sets $B_c(f,t)$ form a basis for a topology on Y^x .

Or

(b) If Y is a closed subset of x, and if x has finite dimension, then prove that Y has finite dimension and $\dim Y \le \dim x$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove the Tietze extension theorem.

17. State and prove the sufficiency of the Nagata Smirnov Metrization theorem.

18. Let $I = [0, 1]$. Prove that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2 .

19. Let X be a space and let (y, d) be a metric space. For the space $\varsigma(x, y)$. Prove that the compact open topology and the topology of compact convergence coincide.

20. State and prove Baire category theorem.

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D–5518

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Pfaffian differential equation.

2. Define orthogonal trajectories of a system of curves on a surface.

3. Find the complete integral of the equation $p^2z^2 + q^2 = 1$.

4. Eliminate the arbitrary function f from the equation $z = f\left(x^2 + y^2\right)$.

5. When we say that a first order partial differential equation is separable?

6. Write down the telegraphy equation.

7. Solve $\left(D^2 - D^1\right)z = 0$.

8. Write down the fundamental idea of Jacobi's method.

9. Define interior Neumann Problem.

10. Write down the d'Alembert solution of the one dimensional wave equation.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

Or

(b) Verify whether the equation :

$z(z + y)dx + z(z + x)dy - 2xydz = 0$ is integrable or not.

12. (a) Find the general integral of the linear partial differential equation $z.(xp - yq) = y^2 - x^2$.

Or

(b) Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.

13. (a) Solve the equation $p^2x + q^2y = z$ using Jacobi's method.

Or

(b) Find the complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, x^2 = 4y$.

14. (a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

Or

(b) Find a particular integral of $(D^2 - D^1)z = 2y - x^2$.

15. (a) Solve the equation $r + s - 2t = e^{x+y}$.

Or

(b) Prove that the solution of a certain Neumann problem can differ from one another by a constant only.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Verify that the equation $yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$ is integrable and find its solution.

17. Find the integral surface of the equation $px + qy = z$ passing through $x + y = 1$ and $x^2 + y^2 + z^2 = 4$.

18. Find the complete integral of $q = (z + px)^2$.

19. Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

20. A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve $y = k \sin^3\left(\frac{\pi x}{l}\right)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at any time t.

D–5519

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Radius of Curvature of a Curve.

2. Define an involute.

3. Write down the equation of the osculating plane at a point of inflexion.

4. State the fundamental existence theorem for space curves.

5. State the intrinsic properties.

6. Give an example for a surface.

7. Write the expression for Geodesic Curvature Kg.

8. When will you say that the vector is called the geodesic vector of the curve?

9. What is meant by the characteristic line?

10. What do you mean by osculating developable of the curve?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Calculate the curvature and torsion of the cubic curve given by $\bar{r} = (u, u^2, u^3)$.

Or

(b) Derive the equation of involute.

12. (a) Find the osculating circle at p, a point to the curve.

Or

(b) Find the area of the Anchor Ring.

13. (a) Prove that the metric is invariant under a parameter transformation.

Or

(b) Enumerate Isometric correspondence.

14. (a) Derive the Canonical equations for geodesics.

Or

(b) Show that a curve on surface in Geodesic if and only if its Gaussian Curvature vector is zero.

15. (a) Show that the characteristics point of the plane u is determined by the equations $\bar{r}.\bar{a} = p$, $\bar{r}.\dot{\bar{a}} = \dot{p}$, $\bar{r}.\ddot{\bar{a}} = \ddot{p}$.

Or

(b) Enumerate the polar developable.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the intrinsic equations of the curve given by $x = ae^u \cos u, y = ae^u \sin u, z = be^u$.

17. Prove that $\tau = \frac{[\bar{r}'\,\bar{r}''\,\bar{r}''']}{|\bar{r}'\times\bar{r}''|^2} = \frac{[\dot{\bar{r}}\,\ddot{\bar{r}}\,\dddot{\bar{r}}]}{|\dot{\bar{r}}\times\ddot{\bar{r}}|^2}$

18. Find the surface of revolution which is isometric with the region helicoid.

19. Prove that $K_g = \frac{1}{H\dot{s}^3}\left(\frac{\partial T}{\partial \dot{u}}V(t) - \frac{\partial T}{\partial \dot{v}}U(t)\right)$.

20. Derive the Rodrigues formula.

D–5520

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Explain triple operation.
2. Define a cut and the cut capacity in a network.
3. Define purchasing cost.
4. What is the difference between PERT and CPM?
5. Define the effective lead time.
6. Define two-person zero-sum game.
7. Write down the Floyd's algorithm.
8. Explain Kendall's notation.
9. Define the transition-rate diagram.
10. What is separable programming?

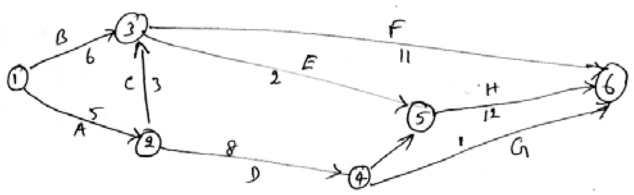
PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain Three-Jug puzzle with an illustration.

Or

(b) Determine the critical path for the project network.



All the durations are in days

12. (a) Solve the game whose payoff matrix is given by

Player B

Player A $\begin{bmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$

Or

(b) Solve the following game and determine the value of the game $\begin{array}{c} \\ A \end{array}\overset{B}{\begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix}}$.

13. (a) Solve the following L.P.P.:

Maximize: $z = 2x_1 + 3x_2 + 10x_3$ subject to the constraints
$x_1 + 2x_3 = 0;\ x_2 + x_3 = 1;\ x_1, x_2, x_3 \geq 0$

Or

(b) What are the total and free floats of a critical activity?

14. (a) Enumerate the no-setup model.

Or

(b) Show, how the following problem can be made separable

Maximize: $z = x_1, x_2 + x_3 + x_1x_3$

Subject to $x_1, x_2 + x_3 + x_1x_3 \leq 0$

$x_1, x_2, x_3 \geq 0$.

15. (a) Use the method of Lagrangian multiples to solve the non-linear programming problem:

Minimize

$z = 2x_1^2, x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to the constraints $x_1 + x_2 + x_3 = 20$

Or

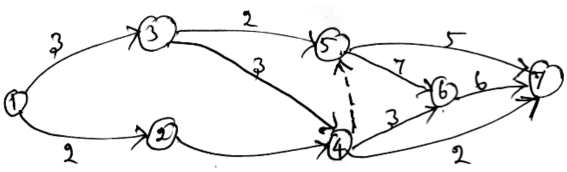
(b) Determine the extreme point of the function

$f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Find the critical path for the following project network:



17. Using the bounded variable technique, solve the following L.P.P.:

Maximize: $z = 4x_1 + 10x_2 + 9x_3 + 11x_4$

Subject to the constraints :
$2x_1 + 2x_2 + 2x_3 + 2x_4 \le 5$
$48x_1 + 80x_2 + 160x_3 + 240x_4 \le 257,$
$0 \le x_j \le 1$ for $i = 1, 2, 3, 4.$

18. Use two-phase revised simplex method to solve the L.P.P:

Minimize : $z = 3x_1 + x_2$ Subject to the constraints

$x_1 + x_2 \ge 1$
$2x_1 + 3x_2 \ge 2,\ x_1,\ x_2 \ge 0$

19. Solve the non-linear programming problem:

Minimize: $z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$

Subject to the constraints $x_1 + x_2 + x_3 = 11.$

20. Solve the quadratic programming problem:

Maximize : $z = 2x_1 + x_2 - x_1^2$ Subject to the constraints.

$2x_1 + 3x_2 \le 6,\ 2x_1 + x_2 \le 4;\ x_1,\ x_2 \ge 0.$

D–5521

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is meant by divisibility? Give an example.

2. When will you say a number is composite? Give an example.

3. If a prime p does not divide a, then prove that $(p,a)=1$.

4. State the Euler totient function $\varphi(n)$.

5. Define Dirichlet convolution.

6. If $a = b(\text{mod}\, m)$ and if $0 \leq |b-a| < m$, then prove that $a = b$.

7. State the Legendre's identity.

8. State the Wilson's theorem.

9. What are the quadratic residue and non-residues mod13?

10. Write down the diophantic equations with an example.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any integer x, Prove that $(a,b) = (b,a) = (a,-b) = (a,b+ax)$.

Or

(b) Prove that if $2^n + 1$ is prime, then n is a power of 2.

12. (a) Let f be multiplicative. Prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n)f(n)$,, for all $n \geq 1$.

Or

(b) If $n \geq 1$, prove that $\phi(n) = \sum_{d/n} \mu(d)\frac{n}{d}$.

13. (a) Prove that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$.

Or

(b) State and prove that Euler's summation formula.

14. (a) Solve the congruence $25x \equiv 15(\bmod 120)$.

Or

(b) State and prove Wolstenholme's theorem.

15. (a) Show that, the Legendre's symbol (n/p) is a completely multiplicative function of n.

Or

(b) Prove that, for any prime p all the coefficients of the polynomial $f(x) = (x-1)(x-2)....(x-p+1) - x^{p-1} + 1$ are divisible by p.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove the Euclidean algorithm.

17. If $2^n - 1$ is prime, then prove that n is a prime.

18. Derive Dirichlet's asymptotic formula.

19. State and prove Lagrange theorem.

20. State and prove the quadratic reciprocity law.

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D–5522

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Stochastic processes.

2. Define order of a Markov chain.

3. State the stochastic matrix.

4. Define diffusion equation.

5. Is Wiener process, a Gaussian process? Justify your answer.

6. Define sample paths.

7. Define Markov renewal branching process.

8. State the Bellman–Harris process.

9. What is meant by Poisson queue?

10. Define birth and death process.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If state j is persistent non-null, then prove that, as $n \to \infty$.

(i) $p_{jj}^{(nt)} \to t/\mu_{jj}$ when state j is periodic with period t and

(ii) $p_{jj}^{(n)} \to 1/\mu_{jj}$ when state j is a periodic

Or

(b) Let $\{x_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution $\Pr(X_0 = i) = \frac{1}{3}, i = 0,1,2$.

Find:

(i) $\Pr(X_2 = 2, x_1 = 1 | x_0 = 2)$

(ii) $\Pr(X_2 = 2, x_1 = 1, x_0 = 2)$.

12. (a) A coin is tossed, p being the probability of head in a toss. Let $\{x_n, n \geq 1\}$ have the two states 0 or 1 according as the accumulated number of heads and tails in n tosses are equal or unequal. Show that the states are transient when $p \neq 1/2$, and persistent null when $p = 1/2$.

Or

(b) Prove that the internal between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter τ has a negative exponential distribution with near $1/\tau$.

13. (a) Let $\{X(t), t \geq 0\}$ be a Wiener process with $\mu = 0$ and $x(0) = 0$. Find the distribution of T_{a+b}, for $0 < a < a + b$.

Or

(b) If $X(t)$, with $X(0)$ and $\mu = 0$, is a Wiener process, show that $Y(t) = \sigma \times (t/\sigma^2)$ is also a Wiener process. Find its covariance function.

14. (a) Find the waiting time density and expected waiting time for $M/M(1,b)/1$ model.

Or

(b) Prove that $P_n(s) = P_{n-1}(P(s))$

15. (a) Derive the Fokker–Planck equation.

Or

(b) Find the moments of the distribution of the waiting time *T*.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Ergodic theorem.

17. Find the differential equation of pure death process. If the process starts with *i* individuals, then find the mean and variance of the number $N(t)$ present at time *t*.

18. If $m = 1, \sigma^2 < \infty,$ then prove that

(a) $\lim_{n\to\infty} n \Pr(x_n > 0) = 2/\sigma^2$

(b) $\lim_{n\to\infty} E\left[\frac{X_n}{n} \Big/ x_n > 0\right] = \frac{\sigma^2}{2}$

(c) $\lim_{n\to\infty} \Pr\left(\frac{X_n}{n} > u \Big/ X_n > 0\right) = \exp\left(\frac{-2u}{\sigma^2}\right), u \geq 0.$

19. Prove that the generating function

$F(t, s) = \sum_{k=0}^{\infty} \Pr\{X(t) = k\} s^k$ of an age-dependent branching process $\{X(t), t \geq 0\};$ $x(0) = 1$ satisfies the integral equation $F(t, s) = [1 - G(t)]s + \int_0^t [F(t-u, s]dG(u).$

20. Derive Pollaczek-Khinchine formula.

D–5523

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

GRAPH THEORY

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define an induced sub graph of a graph with an example.

2. Give an example of a closed walk of even length which does not contain a cycle.

3. Define Ramsey numbers.

4. Define centre of a tree.

5. Define chromatic number of a graph.

6. What is meant by a critical graph?

7. State the Kuratowski graph.

8. Define a critical graph. Give an example.

9. Define directed graph.

10. Define minimum in degree and maximum outdegree.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain adjacency matrix and incidence matrix with examples.

Or

(b) If k-regular bipartite graph with $k > 0$ has bipartition (x, y), then prove that $|x| = |y|$.

12. (a) Prove that $C(G)$ is well defined.

Or

(b) If G is a k-regular bipartite graph with $k > 0$ then prove that G has a perfect matching.

13. (a) State and prove the Berge theorem.

Or

(b) Let G be a connected graph that is not an odd cycle, prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast two.

14. (a) Prove that the complete graph k_5 is non planar.

Or

(b) Prove that every critical graph is a block.

15. (a) If two digraphs are isomorphic then prove that the corresponding vertices have the same degree pair.

Or

(b) If a digraph D is strongly connected then prove that D contains a directed closed walk containing all its vertices.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that $\tau(k_n) = n^{n-2}$.

17. State and prove Brook's theorem.

18. Prove that every planar graph is 5- colourable.

19. State and prove Euler's theorem.

20. Prove that the edges of a connected graph G can be oriented so that the resulting digraph is strongly connected if and only if every edge of G is contained in atleast one cycle.

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D–5524

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define convex set. Give an example.

2. Define the normed dual.

3. Define bounded linear map.

4. Define inner product space.

5. Let X, Y be metric spaces. If $f : X \to Y$ is continuous and $g : X \to Y$ is closed, then prove that $f + g : X \to Y$ is closed.

6. Define completely continuous linear transformation.

7. Define adjoint and self-adjoint operators.

8. Show that the vector space of all null sequences c_0 is not reflexive.

9. Define weak convergence in a Hilbert space.

10. State the closed graph theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (X,d) and (Y,d') are metric spaces and $f : X \rightarrow Y$, then prove that f is continuous if and only if $f^{-1}(F)$, where F is any closed set in Y, is closed set in X.

Or

(b) If a Cauchy sequence (x_n) in a normed space has a convergent subsequence (x_{n_k}), then prove that (x_n) is convergent.

12. (a) Prove that the linear functional f on the normed linear space X is bounded if and only if it is continuous.

Or

(b) If x and y are two elements in a normed linear space X, then prove that $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.

13. (a) Prove that the Schwarz inequality $\|< x, y >\| \leq < x, x >^{1/2} . < y, y >^{1/2}$ for all x, y in an inner product space X.

Or

(b) Prove that the inner product is jointly continuous. In particular $y_n \rightarrow y \Rightarrow < x, y_n > \rightarrow < x, y >$.

14. (a) Prove that an orthonormal set in an inner product space is linearly independent.

Or

(b) Let the dual space of the normed linear space X be $\tilde{X}$. Prove that if $\tilde{X}$ is separable then X is separable

15. (a) Let A be a linear transformation on the finite dimensional space X. Prove that A is completely continuous.

Or

(b) Suppose $A : X \rightarrow Y$. If A is completely continuous then prove that the range of A, $R(A)$ is separable.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that all compact sets are countably compact.

17. In the normed linear space X, suppose that $\{x_\alpha\}$ is summable to $x \in X$, where α runs through an index set $\wedge$. Prove that all but a countable number of the x_α must be zero.

18. If A is completely continuous then prove that its conjugate map A' is completely continuous.

19. State and prove Hahn-Banach theorem.

20. State and prove open mapping theorem.

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D–5525

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define simple root.

2. What is meant by rate of convergence of iterative method.

3. Define spectral radius of a matrix A.

4. What is meant by interpolating polynomial $P(x)$?

5. Write the Hermite interpolating polynomial.

6. State any two properties of cubic spline interpolation.

7. Write the general Euler's formula.

8. Write down the formula for numerical differentiation of y with respect to x once.

9. Define mesh points.

10. What is the advantages of Runge-Kutta method.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find a real root $f(x) = x^3 - 5x + 1 = 0$ in the interval $(0,1)$. Calculate the first six iterations only.

Or

(b) Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$. Use initial approximation $P_0 = 0.5, q_0 = 0.5$.

12. (a) Solve the following equations by using Gauss elimination method :

$$x + y + z = 6$$
$$3x + 3y + 4z = 20$$
$$2x + y + 3z = 13$$

Or

(b) Using the Jacobi method find all the eigen values and the corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

13. (a) From the following $f(x, y)$ data, find $f(0.25, 0.75)$ using linear interpolation.

y \ x	0	1
0	1	1.414214
1	1.732051	2

Or

(b) Find the least square approximation of second degree for the following data:

x	–2	–1	0	1	2
$f(x)$	15	1	1	3	19

14. (a) A differentiation rule of the form $hf'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$ where $x_j = x_0 + jh,\ j = 0,1,2,3,4$ is given. Determine the values of α_0, α_1, α_2 and α_3 so that the rule is exact for a polynomial of degree 4 and find an expression for the round-off error in calculating $f'(x_2)$.

Or

(b) Using the following data, find $f'(6.0)$, error = $o(h)$ and $f''(6.3)$, error = $o(h^2)$.

x	6.0	6.1	6.2	6.3	6.4
$f(x)$	0.1750	–0.1998	–0.2223	–0.2422	–0.2596

15. (a) Find the appropriate value of $I = \int_0^1 \frac{\sin x}{x} dx$ using mid point rule and two point open type rule.

Or

(b) Evaluate $\int_0^\infty (3x^3 - 5x + 1) e^{-x}\, dx$ using the Gauss-Laguerre two point formula.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Obtain the complex root of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places. Use the initial approximation to a root as $(x_0, y_0) = (0.25, 0.25)$.

17. Find all the roots of the polynomial equation $x^3 - 4x^2 + 5x - 2 = 0$ using the Graeffe's root squaring method.

18. Given the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
–1	1	–5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

19. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$ using backward Euler method.

20. Solve the initial value problem $u'' = (1 + t^2)\, u$, $u(0) = 1$, $u'(0) = 0$, $t \in [0, 0.4]$ by using Runge-kutta method with $h = 0.2$.

D–5526

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Let A and B be independent events with $P(A) = 0.7, P(B) = 0.2$. Find $P(A \cap B)$.

2. Prove that the expected value of the product of two random variables is equal to the product of their expectations plus their covariance.

3. Define a negative binomial distribution.

4. Write down the moment generating function of $Y = \sum_{i=1}^{n} X_i$.

5. If the p.d.f of X is $f(x) = \begin{cases} 2xe^{-x^2}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$. Determine the p.d.f of $y = x^2$.

6. If the random variable x has a Poisson distribution such that $\Pr(x = 1) = \Pr(x = 2),$, then find $\Pr(x = 4)$.

7. Let $\overline{X}$ be the mean of a random sample of size 5 from a normal distribution with $\mu = 0$ and $\sigma^2 = 125$. Determine c so that $P(\overline{X} < c) = 0.90$.

8. Define covariance in distribution.

9. When we say that a sequence of random variables $X_1, X_2, ...$ converges in probability to a random variable X?

10. Let Zn be $\chi^2(n)$. Find the mean and variance of Zn.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Chebyshev's inequality.

Or

(b) Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of drawing :

(i) two hearts

(ii) A heart on the first draw, a club on the second draw.

12. (a) Let $f(x_1, x_2) = 1/16, x_1 = 1,2,3,4$ and $x_2 = 1, 2, 3, 4$ and zero elsewhere be the joint p.d.f. of X_1 and X_2. Show that X_1 and X_2 are independent.

Or

(b) Let $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ be the p.d.f of X. Find the distribution function and the p.d.f. of $y = \sqrt{x}$.

13. (a) Let X have a Poisson distribution with $\mu = 100$. Use Chebyschev's inequality to determine a lower bound for $P_r(75 < x < 125)$.

Or

(b) If the correlation coefficient ρ of X exists then prove that $-1 \le \rho \le 1$.

14. (a) Let $\overline{X}$ denote the mean of a random sample of size 36 from an exponential distribution with mean 3. Find $P(2.5 \le \overline{x} \le 4)$.

Or

(b) Let X and Y be random variables with $\mu_1 = 1$, $\mu_2 = 4$, $\sigma_1^2 = 4$, $\sigma_2^2 = 6$, $S = \frac{1}{2}$. Find the mean and variance of $z = 3x - 2y$.

15. (a) Let X have the uniform distribution over the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Show that $y = \tan x$ has a Cauchy distribution.

Or

(b) Let $F_n(y)$ be the distribution function of a random variable y_n whose distribution depends upon the positive integer n. Prove that the sequence $y_n, n = 1, 2, \ldots$ converges in probability to the constant c if and only if the limiting distribution of y_n degenerate at $y = c$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let X have the p.d.f $f(x) = \begin{cases} \frac{x+2}{18}, & -2 < x < 4 \\ 0, & \text{elsewhere} \end{cases}$. Find $E(X), E\left[(X+2)^3\right]$ and $E\left[6X - 2(x+2)^3\right]$.

17. Find the m.g.f of a normal distribution and hence find the mean and variance of a normal distribution.

18. Let the random variables X and Y have the joint p.d.f $f(x, y) = \begin{cases} x + y, & 0 < x < 1; 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

    Compute the correlation coefficient of X and Y.

19. Derive the p.d.f of the beta distribution with parameters α and β.

20. State and prove central limit theorem.

D–4468

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

ALGEBRA — I

(CBCS–2018–2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define mappings.

2. Define subgroup. Give an example.

3. Define automorphism of a group.

4. What is meant by normal subgroup?

5. Show that the group of order 21 is not simple.

6. Define an internal direct product of groups.

7. Define a maximal ideal of a ring with an example.

8. If D is an integral domain with finite characteristic, then prove that the characteristic of D is a prime.

9. State unique factorization theorem.

10. Prove that an Euclidean ring posses an unit element.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For all $a \in G$, prove that $H_a = \{x \in G / a \equiv x \bmod H\}$.

Or

(b) Prove that HK is a subgroup of G if and only of $HK = KH$.

12. (a) Prove that, N is a normal subgroup of G if and only if $gN^{-1}g = N$ for every $g \in G$.

Or

(b) Prove that a group of order 9 is abelian.

13. (a) If υ is an ideal of R and $I \in \upsilon$, then prove that $\upsilon = \mathrm{R}$.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field.

14. (a) Let R be a Euclidean ring. Suppose that for $a, b, c \in R$, $a \mid bc$ and $(a, b) = 1$. Then prove that $a \mid c$.

Or

(b) Prove that the mapping $\phi : D \to F$ defined by $\phi(a) = [\alpha, 1]$ is an isomorphism of D into F.

15. (a) State and prove Eisenstein criterion.

Or

(b) If p is a prime number of the form $4n+1$, then prove that $p = a^2 + b^2$ for some integers a and b.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$, respectively then prove that $o(HK) = \dfrac{o(H)o(K)}{o(H \cap K)}$.

17. State and prove Cauchy's theorem.

18. Prove that every finite abelian group is the direct product of cyclic groups.

19. Prove that $J[i]$ is a Euclidean ring.

20. State and prove unique factorization theorem.

D–4469

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define bounded set.

2. Define ordered field. Give an example.

3. Define metric space. Give an example.

4. Define perfect sets. Give an example.

5. State the root tests.

6. Find the radius of convergence of the power series $\sum \frac{2^n}{n^2} z^n$.

7. Define Uniformly continuous functions. Give an example.

8. Let f be defined on $[a,b]$. If f is differentiable at a point $x \in [a,b]$, then show that f is continuous at x.

9. State the intermediate theorem.

10. Define contraction.

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If x and y are complex, prove that $||x|-|y|| \leq |x - y|$.

Or

(b) Prove that the set of all rational numbers in countable.

12. (a) Show that every k-cell is compact.

Or

(b) Prove that a finite point set has no limit point.

13. (a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Prove that $f(x)$ is compact.

Or

(b) If f is continuous mapping of a metric space X into a metric space Y and if E is connected subset of X, prove that $f(E)$ is connected.

14. (a) State and prove generalized mean value theorem.

Or

(b) State and prove Cantor intersection theorem.

15. (a) State and prove intermediate value theorem for derivatives.

Or

(b) State and prove chain rule for differentiation.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. For every real $x > 0$ and every integer $n > 0$, prove that there is one and only real y such that $y^n = x$.

17. Prove that $\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

18. State and prove Bolzano-Weierstrass theorem.

19. State and prove Taylor's theorem.

20. State and prove the inverse function theorem.

D–4470

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Find all real valued solutions of the equation $y'' - y = 0$.

2. Define linearly independent and linearly dependent functions.

3. Find two linearly independent solutions of $y'' - \frac{1}{4x^2} y = 0$, $x > 0$.

4. Find the singular points of the equation $x^2 y'' + (x + x^2) y' - y = 0$ and determine whether they are regular singular points or not.

5. Write any two legendre polynomials.

6. Define indicial polynomial.

7. State the Bessel function.

8. Find the integrating factor of the function $\cos x \cos y dx - 2 \sin x \sin y \, dy = 0$.

9. State the Lipschitz condition.

10. State the local existence theorem for the initial value problem $y' = f(x, y), y(x_0) = y_0$ on z.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the uniqueness theorem for initial value problem.

Or

(b) Find the solutions of the initial value problem $y'' + 10y = 0$, $y(0) = \pi$, $y' = \pi^2$.

12. (a) Obtain two linearly independent solutions of $x^2 y'' + 2x^2 y' - 2y = 0$.

Or

(b) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all the solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

13. (a) Find all solutions of the equation $x^2 y'' + xy' - 4\pi y = x$ for $x > 0$.

Or

(b) Find the two independent solution of the equation $(3x-1)^2 y'' + (9x-3)y' - 9y = 0$ for $x > \dfrac{1}{3}$.

14. (a) Show that –1 and +1 are regular singular points for the legendre equation $(1-x^2)y''-2xy'+\alpha(\alpha+1)y=0$.

Or

(b) Find the first four successive approximate $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y'=1+xy,\ y(0)=1$.

15. (a) Let f be a continuous and satisfy a Lipschitz condition on R. If ϕ and ψ are two solutions $y'=f(x,y)$, $y(x_0)=y_0$ on an interval I containing x_0, then prove that $\phi(x)=\psi(x)$ for all x in I.

Or

(b) Find the solution ϕ of $y''=1+(y')^2$ which satisfies $\phi(0)=0, \phi'(0)=0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let ϕ_1 and ϕ_2 be two solutions of $L(y)=0$ on an interval I and let x_0 be any point of I. Then prove that ϕ_1 and ϕ_2 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0)\neq 0$.

17. Determine the real valued solutions of

(a) $y^{(4)}-y=0$ and

(b) $y''-2iy'-y=0$.

18. Derive Bessel's function of first kind of order $\alpha, J_\alpha(x)$.

19. Find all solutions of

(a) $x^2 y'' + xy' - 4y = x$ and

(b) $x^2 y'' + xy' + 4y = 1$, $|x| > 0$.

20. Let M, N be two real valued functions which have continuous first order partial derivatives on some rectangle $R : |x - x_0| \leq a$, $|y - y_0| \leq b$. Prove that the equation $M(x, y) + N(x, y)y' = 0$ in exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

D–4471

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a function. Give an example.

2. State the axiom of choice.

3. Define basis for a topology.

4. State the Hausdorff space.

5. Define continuous function.

6. Is the rational Q connected? Justify your answer.

7. Define compact space. Give an example.

8. Define limit point compact. Give an example.

9. Define normal space.

10. State the Urysohn lemma.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Or

(b) Prove that the set $\mathcal{S}(Z_+)$ of all subsets of Z_+ is uncountable.

12. (a) Prove that the lower limit topology τ' on $\mathbb{R}$ is strictly finer than the standard topology τ.

Or

(b) Prove that every finite point set in a Hausdorff space is closed.

13. (a) State and prove the pasting lemma.

Or

(b) Prove that the union of a collection of connected sets that have a point in common is connected.

14. (a) State and prove the intermediate value theorem.

Or

(b) Prove that the image of a compact space under a continuous map is compact.

15. (a) Prove that every metrizable space is normal.

Or

(b) Prove that every well ordered set X is normal in the order topology.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that every non-empty subset of Z_+ has a smallest element.

17. Let $\bar{d}(a,b) = \min\{|a-b|, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points of $\mathbb{R}^w$, define $D(x,y) = l.u.b\left\{\frac{\bar{d}(x_1, y_1)}{i}\right\}$. Prove that D is a metric that induces the product topology on $\mathbb{R}^w$.

18. Prove that every compact subset of a Hausdorff space is closed.

19. Prove that the product of finitely many compact space is compact.

20. State and prove Urysohn metrization theorem.

D–4472

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. When will you say that a vector space V is finite dimensional?

2. Define a basis of a vector space.

3. Define internal direct sum.

4. What is meant by the annihilator of W?

5. Define algebraic number.

6. Write short notes on the splitting field over F.

7. What is meant by the Galois group of $f(x)$?

8. Define a characteristic vector of T.

9. Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

10. Define Hermitian and skew-Hermitian.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If A and B are subspaces of V, prove that $(A+B)/B$ is isomorphic to $A/(A \cap B)$.

Or

(b) Prove that $L(S)$ is a subspace of V.

12. (a) Prove that $W^{\perp}$ is a subspace of V.

Or

(b) If $u, v \in V$ then prove that $|\langle u, v\rangle| \leq \|u\| \cdot \|v\|$.

13. (a) If V is finite dimensional and W is a subspace of V, then prove that $\hat{W}\, W$ is isomorphic to $\hat{V} / A(W)$ and $\dim A(W) = \dim V - \dim W$.

Or

(b) State and prove Bessel's inequality.

14. (a) If $\alpha_1, \alpha_2, \alpha_3$ are the roots of the cubic polynomial $x^3 + 7x^2 - 8x + 3$, find the cubic polynomial whose roots are $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}$.

Or

(b) If $T \in A(V)$ is such that $\langle vT, v\rangle = 0$ for all $v \in V$, then prove that $T = 0$.

15. (a) If N is normal and $AN = NA$, then prove that $AN^* = N^*A$.

Or

(b) Prove that the normal transformation N is

(i) Hermition if and only if its characteristic roots are real.

(ii) Unitary if and only if its characteristic roots are all of absolute value 1.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If S, T are subsets of V, then prove that

(a) $S \subset T$ implies $L(S) \subset L(T)$

(b) $L(S \cup T) = L(S) + L(T)$

(c) $L(L(S)) = L(S)$

17. If V and W are of dimensions m and n_1 respectively, over F, then prove that $Hom(V, W)$ is of dimension mn over F.

18. Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

19. State and prove the fundamental theorem of Galoi's theory.

20. Prove that the multiplicative group of non-zero elements of a finite field is cyclic.

D–4473

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define the upper and lower Riemann integrals of f over $[a, b]$.

2. Define an refinement of a partition.

3. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

4. Define an equicontinuous function on a set.

5. Prove that $\lim_{h \to 0} \frac{E + (z + h) - E(z)}{h} = E'(z)$.

6. Define measurable set.

7. If A and B are two sets in M with $A \subset B$, then prove that $mA \leq mB$.

8. Define a simple function.

9. Define Lebesgue integral.

10. Define an integrable function over the measurable set.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\int_{-a}^{b} f(dx) \leq \int_{a}^{-b} f\, d\, x$.

Or

(b) Suppose f is a bounded real on $[a, b]$ and $f^2 \in R$ on $[a, b]$. Does it follow that $f \in R$?

12. (a) State and prove integration by parts theorem.

Or

(b) State and prove Cauchy criterion for uniform convergence.

13. (a) Prove that every non-constant polynomial with complex coefficients has a complex root.

Or

(b) State and prove Parseval's theorem.

14. (a) If E_1 and E_2 are measurable, then prove that $E_1 \cup E_2$ is measurable.

Or

(b) If f is a measurable function and $f = g$, then prove that g is measurable.

15. (a) If $f \in \mathcal{L}(\mu)$ on E, then prove that $|f| \in \mathcal{L}(\mu)$ on E, and $\left|\int_E f \, d\mu\right| \leq \int_E |f| \, d\mu$.

Or

(b) State and prove Fatou's lemma.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove uniform convergence theorem.

17. State and prove the Stone-Weierstrass theorem.

18. If $x > 0$ and $y > 0$, then prove that $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma y}{\Gamma(x+y)}$

19. State and prove Egoroff's theorem.

20. State and prove Lebessgue's dominated convergence theorem.

D–4474

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

TOPOLOGY – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the countable intersection property.

2. Define compactification in a space.

3. Define locally compact. Give an example.

4. Define a G_δ-set. Give an example.

5. Is two compactifications equivalent? Justify your answer.

6. What is meant by the point open topology?

7. Define the evaluation map.

8. Define compact convergence topology.

9. Define a Baire space. Give an example.

10. Define general position in $\mathbb{R}^N$.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a Hausdorff space. Prove that X is locally compact if and only if given x in X and a neighborhood $\mho$ of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset \mho$.

Or

(b) If X is completely regular and non-compact, then prove that $\beta(X)$ is not metrizable.

12. (a) Prove that every closed subspace of a paracompact space is paracompact.

Or

(b) Prove that the product of completely regular space is completely regular.

13. (a) Let X be completely regular. Show that X is connected if and only if $\beta(X)$ is connected.

Or

(b) Let X be normal and let A be closed G_δ-set in X. Prove that there is a continuous functions $f : X \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) If X is locally compact or X satisfies the first countability axiom, then prove that X is completely regular.

Or

(b) Show that the sets $B_C(f, \varepsilon)$ form a basis for a topology of Y^X.

15. (a) Prove that every open subset of a Baire space is a Baire space.

Or

(b) Prove that every compact subset of $\mathbb{R}^N$ has topological dimension at most N.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove the Tychonoff theorem.

17. State and prove the stone cech compactification theorem.

18. Let $I = [0,1]$. Prove that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2.

19. State and prove Ascoli's theorem.

20. Let $X = Y \cup Z$, where Y and Z are closed sets in X having finite topological dimensions. Prove that $\dim X = \max\{\dim Y, \dim Z\}$.

D–4475

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define Pfaffian differential equation.

2. What is meant by orthogonal equation trajectories?

3. Define self-orthogonal.

4. Write the form of the linear PDE of order one.

5. Solve : $a(p+q)=z$.

6. What is the general solution of $Pp + Qq = R$?

7. Show that the differential equations $\frac{\partial Z}{\partial x} = x^2 - ay$ and $\frac{\partial z}{\partial y} = y^2 - ax$ are compatible.

8. Solve $\left(2D^2 - 5DD' + 2D'^2\right)z = 0$.

9. Solve : $\frac{\partial^2 z}{\partial x^2} = \frac{1}{a} - xy$.

10. Write down the interior Neumann boundary value problem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Solve : $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}$.

Or

(b) Solve : $\frac{x\,dx}{y^3 x - 2x^4} = \frac{dy}{2y^4 - x^3 y} = \frac{dz}{9z\left(x^3 - y^3\right)}$.

12. (a) Show that

$(yz + 2x)\,dx + (zx - 2z)\,dy + (xy - 2y)\,dz = 0$ is integrable.

Or

(b) Find the differential equation of the set of all right circular cones whose axes coincide with z-axis.

13. (a) Find the equation of surface satisfying $4yz\,p + q + 2y = 0$ and passing through $y^2 + z^2 = 1,\; x + z = 2$.

Or

(b) Find the family orthogonal to

$\phi\left[z\,(x + y)^2, x^2 - y^2\right] = 0$.

14. (a) Solve $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = xyz$.

Or

(b) Find the complete integral of $q = px + p^2$.

15. (a) Reduce $\frac{\partial^2 z}{\partial x^2} = (1+y)^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

Or

(b) Derive D'Alembert's solution to the one-dimensional wave equation.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that the orthogonal trajectories of the family of conics $y^2 - x^2 + 4xy - 2cx = 0$ consists of a family of cubics with the common asymptote $x + y = 0$.

17. Solve $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$.

18. Find the complete integral of $xp^2 - ypq + y^3 q - y^2 z = 0$.

19. Solve : $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y} + xy$.

20. A taut string of length $2l$ is fastened at both ends. The mid-point of the string is taken to a height b and then released from reset in that position. Find the displacement of the string at time t.

D–4476

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is meant by binomial line?
2. Define an osculating circle.
3. What is meant by point of inflexion?
4. Define evolute.
5. Write short notes on the general helicoid.
6. Explain geodesic parallels.
7. Write the canonical equations for geodesies.
8. Define the osculating development of the curve.
9. Define principal curvature.
10. Define developable.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the equation of the osculating plane of the curve given by

$$\vec{x} = \{a \sin u + b \cos u, a \cos u + b \sin u, c \sin 2u\}.$$

Or

(b) With the usual notations, prove that $\left[\vec{\gamma}', \vec{\gamma}'', \vec{\gamma}'''\right] = k^2 \lambda$.

12. (a) For any general helix c prove the following relation between its curvature and that of the plane curve c_1 obtained by projecting c on a plane orthogonal to its axis $k = k_1{}^2 \sin^2 \alpha$.

Or

(b) Find the coefficients of direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

13. (a) Show that every helix on a cylinder is a geodesic.

Or

(b) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma \rho') + \frac{\rho}{\sigma} = 0$.

14. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z = a$ constant.

Or

(b) Find the geodesic curvature of the parametric curve $v = c$, where c is a constant.

15. (a) Explain an Umbilic point.

Or

(b) Write a short note on “Dupins Indicatrix”.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive Serret–Frenet formula.

17. Show that the intrinsic equations of the curve given by $x = ae^u \cos u, y = ae^u \sin u, z = be^u$ are

$$k = \frac{a\sqrt{2}}{\left(2a^2 + b^2\right)^{1/2}}, \frac{1}{s}; I = \frac{b}{\left(2a^2 + b^2\right)^{1/2}}; \frac{1}{s}.$$

18. Find a surface of revolution which is isometric with a regions of the right helicoid.

19. State and prove Gauss–Bonnet theorem.

20. Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve form a developable.

D–4477

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define oriented arc.

2. What is meant by connected network?

3. Define critical path.

4. Define optimal feasible solution.

5. Explain bounded variables.

6. Define two person–zero sum game.

7. Explain the sensitivity analysis.

8. Define concave and convex function.

9. Define separable function – Give an example.

10. Define quadratic programming model.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain Dijkstra's algorithm to find the shortest route.

Or

(b) Construct the network diagram comprising activities $B, C, \ldots\ldots Q$ and N such that the following constraints are satisfied $B < E, F$; $C < G, L$; $E, G < H$; $L, H < I$; $L < M$; $H < N$, $H < J$; $I, J < P$; $P < Q$ the notation $x < y$ means that the activity x must be finished before y can begin.

12. (a) Explain EOQ with price breaks.

Or

(b) Explain the maximal flow problem.

13. (a) Solve the LPP by using simplex method.

Maximize : $z = 30x_1 + 20x_2$

Subject to

$$10x_1 + 8x_2 \leq 800$$
$$x_1 \leq 60$$
$$x_2 \leq 75$$
$$x_1, x_2 \geq 0$$

Or

(b) Solve the following game

		Player B			
		1	2	3	4
Player A	1	8	2	9	5
	2	6	5	7	18
	3	7	3	–4	10

14. (a) Solve by Jacobian method

Maximize $z = 2x_1 + 3x_2$

Subject to

$$x_1 + x_2 + x_3 = 5$$
$$x_1 - x_2 + x_9 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Or

(b) Solve the non linear programming problem by using Lagcangian multipliers

Minimize $z = x_1^2 + x_2^2 + x_3^2$

Subject to $4x_1 + x_2^2 + 2x_3 = 14$.

15. (a) Use the Kuhn–Tucker conditions to solve the non linear programming problem

Maximize $z = 2x_1^2 + 12x_1 x_2 - 7x_2^2$

Subject to $2x_1 + 5x_2 \leq 98$.

Or

(b) Solve the following quadratic problem

Minimize $z = x_1^2 - 2x_1 x_2 + 2x_2^2 - 2x_1 - 5x_2$

Subject to

$$2x_1 + 3x_2 \leq 20$$
$$3x_1 - 5x_2 = 4$$
$$x_1 \geq 0, \text{ and } x_2 \geq 0$$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Construct the dual of the problem

Minimize $z = 3x_1 + 10x_2 + 2x_3$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

17. The payoff matrix of a two person zero–sum game is :

		Player B		
		B_1	B_2	B_3
Player A	A_1	1	2	1
	A_2	0	-4	-1
	A_3	1	3	-2

Determine the number of saddle points and the corresponding optimal solutions.

18. Reduce the game by dominance property and solve it

		Player B				
		1	2	3	4	5
Player A	I	1	3	2	7	4
	II	3	4	1	5	6
	III	6	5	7	6	5
	IV	2	0	6	3	1

19. Derive the optimal solution from the Kuhn–Tuker conditions for the problem.

Minimize : $z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to

$x_1 + 3x_2 \leq 6$

$5x_1 + 2x_2 \leq 10$

and $x_1 \geq 0, x_2 \geq 0$.

20. Solve the quadratic programming problem.

Minimize : $f(x_1, x_2) = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$

Subject to

$2x_1 + x_2 \leq 6$

$x_1 - 4x_2 \leq 0$

$x_1 \geq 0, x_2 \geq 0$

D–4478

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State Euclid's Lemme.

2. Define prime number and composite number. Give example.

3. State the Euler totient function $\phi(n)$.

4. When will you say the functions are division functions?

5. State the little fermat theorem.

6. Determine whether -10_4 is e quadratic residue or non-residue of the prime 997.

7. Prove that $[-x]=\begin{cases} -[x] & \text{if} \quad x=[x] \\ -[x]-1 & \text{if} \quad x \neq [x] \end{cases}$.

8. State Wilson's theorem.

9. What are the quadratic residues and non residues $\bmod 13$?

10. Write the Diophantive equations.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any positive integer m, prove that

$(ma, mb) = m(a, b)$.

Or

(b) Find the values of x and y to satisfy

$243x + 198y = 9$.

12. (a) If $x \equiv y \pmod{m}$ then prove that $(x, m) = (y, m)$.

Or

(b) State and prove the division algorithm.

13. (a) If $x \geq 1$ then prove that $\sum_{n > x} \frac{1}{n^s} = 0(x^{1-s})$ if $s > 1$.

Or

(b) If $x \geq 1$, prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + 0\left(\frac{1}{x}\right)$.

14. (a) If a prime p does not divide a, then prove that

$a^{p-1} = 1 \pmod{p}$.

Or

(b) Prove that an arithmetic function f has a multiplicative inverse if and only if $f(1) \neq 0$. Also prove that, if an inverse exists it is unique.

15. (a) State and prove Gauss lemma.

Or

(b) If Q is odd and $Q > 0$, then prove that

$\left(\frac{-1}{Q}\right) = (-1)^{(Q-1)/2}$ and $\left(\frac{2}{Q}\right) = (-1)^{(Q^2-1)/8}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove the fundamental theorem of arithmetic.

17. Show that $a(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.

18. Let f be multiplicative prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) = f(n)$ for all $n \geq 1$.

19. State and prove Euler's summation formula.

20. State and prove that quadratic reciprocity law.

D–4479

Sub. Code
31134

DISTANCE EDUCATION

B.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Stochastic processes.

2. When do you say that a Markov chain have countable state space?

3. Define order of a Markov chain.

4. Define diffusion processes.

5. Write down the equation of motion of a Brownian particle.

6. Define generating function.

7. What is meant by traffic intensity?

8. What is meant by rate equality principle?

9. Define busy period.

10. State the Erlang's loss formula. $B\left(s, \lambda/\mu\right)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain random walk between two barriers.

Or

(b) Let $\{x_n, n \geq 0\}$ be a Markov chain with three states 0,1,2 and with transition matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution.

$\Pr\{x_i = i\} = 1/3, i = 0,1,2.$. Find

$\Pr(x_1 = 1 | x_0 = 2), \Pr(x_2 = 2, x_1 = 1 | x_0 = 2)$ and $\Pr(x_3 = 1, x_2 = 2, x_1 = 1, x_0 = 2)$

12. (a) If $\{N(t)\}$ is a poisson process and $s < t$ then prove that $\Pr\{N(s) = K \mid N(t) = n\} = \binom{n}{k}\left(\frac{s}{t}\right)^k \left(1 - \left(\frac{s}{t}\right)\right)^{n-k}$.

Or

(b) Explain Birth-immigration process.

13. (a) If $x(t)$ with $x(0)$ and $\mu = 0$, is a Wiener process and $0 < s < t$, show that for at least one τ satisfying $s \leq \tau < t, \Pr\{x(\tau) = 0\} = \left(\frac{2}{\pi}\right)\cos^{-1}\left(\left(\frac{s}{t}\right)^{1/2}\right)$.

Or

(b) Show that for an irreducible ergodic process

$$\lim_{t \to \infty} \frac{M(k, j, t)}{M(h, i, t)} \to \frac{v_j}{v_i}.$$

14. (a) Explain waiting time in the queue.

Or

(b) Find the moments of the waiting time T.

15. (a) Derive Pollaczek – Khinchine formula.

Or

(b) Show that the distribution of $N(t)$ is poisson with mean $\lambda at = \lambda \int_{o}^{t} \{1 - B(u)\} du$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that the p.g.f. of a non-homogeneous process $\{N(t); t \geq 0\}$ is $Q(s,t) = \exp\{m(t)(s-1)\}$.

Where $m(t) = \int_{0}^{1} \lambda(x) dx$ is the expectation of $N(t)$.

17. Derive the forward and backward Chapman – Kolmogorov differential equations.

18. The number of accident in a town follows a poisson process with a mean 2 per day and the number x_i of people involved in the i^{th} accident has the distribution (Independent) $\Pr\{x_i = k\} = \frac{1}{2^k}, k \geq 1$. Find the mean and the variance of the number of people involved in accidents per week.

19. Find the distribution of the number (Q) in the queue in steady state in an $M/M/1$ queue with rates λ, μ. Find $E(Q)$ and $Var(Q)$. Verify that $E(Q) = \lambda E(W_Q)$. Also find $Var(W_Q)$.

20. Derive the Erlang's second formula.

D–4480

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define regular graph. Give an example.

2. Define a path with an example.

3. Define a cut vertex of a graph.

4. Define tree. Give an example.

5. Define clique of a graph.

6. What is meant by edge chromatic number of a graph?

7. Define a dual graph.

8. Define planar and non-planar graph.

9. Define in degree and out degree of a vertex.

10. Define directed walk.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every cubic graph has an even number of points.

Or

(b) With usual notations prove that $\alpha + \beta = p$.

12. (a) With usual notations prove that $K \leq K' \leq \delta$.

Or

(b) Define a block of a graph with an example. If G is a block with $v \geq 3$, then prove that any two edges of G lie on a common cycle.

13. (a) State and prove Berge theorem.

Or

(b) Prove that a graph G with atleast two vertices is biparticle if and only if all its cycles are of even length.

14. (a) If G is a tree with n vertices, $n \geq 2$, then prove that $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$.

Or

(b) State and prove Dirac theorem.

15. (a) Define a planar graph. Show that the complete graph K_5 is non-planar.

Or

(b) If two diagrams are isomorphic then prove that the corresponding vertices have the same degree pair.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove character theorem.

17. Prove that every uniquely n-colourable graph is $(n-1)$ connected.

18. State and prove Vizing's theorem.

19. (a) Show that $K_{3,3}$ is non-planar.

(b) If G is a simple graph, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

20. Prove that a weak diagraph D is Eulerian if and only if every vertex of D has equal in degree and out degree.

D–4481

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Banach space.

2. Define Compact set.

3. Define bounded linear functional.

4. Define inner product space. Give an example.

5. Define project map.

6. Define orthogonal set.

7. Define self-adjoint operator.

8. Define closed convex hull.

9. Define convex functional.

10. State the closed graph theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (X,d) and (Y,d') are metric spaces where $f: X \to Y$ then prove that f is continuous at the point x if and only if for every sequence $\{x_n\}$ converging to x, $f(x_n) \to f(x)$.

Or

(b) Let (X,d) be complete and let A be totally bounded. Prove that A is relatively compact.

12. (a) State and prove Zorn's lemma.

Or

(b) If X is a finite dimensional linear space, then prove that all linear functionals are bounded.

13. (a) Let X be a real inner product space and let $x, y \in X$. Prove that $< x, y >= \frac{1}{4}\left[\|x+y\|^2 - \|x-y\|^2\right]$.

Or

(b) Prove that the eigen vectors associated with distinct eigen values of a self-adjoint linear transformation are orthogonal.

14. (a) If X is an inner product space, prove that the inner product $< x, y >$ is a continuous mapping $X \times X$ into F.

Or

(b) Let X be an inner product space and let $A = \{x_\alpha\}_{\alpha \in \wedge}$, be an orthonormal set in X. Prove that space if for any $x \in X$, $\|x\|^2 = \sum_{\alpha} |< x, x_\alpha >|^2$, then A is complete.

15. (a) If T is an operator on H for which $< Tx, x >= 0$ for all x, then prove that $T = 0$.

Or

(b) Prove that a bounded linear operator T on H is unitary if and only if it is a linear isometry of H onto itself.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that all compact sets are countably compact.

17. Let $\tilde{X}$ denote the normed linear space of all bounded linear functionals over the normed linear space X. Prove that $\tilde{X}$ is a Banach space.

18. State and prove Riesz theorem.

19. State and prove uniform boundedness theorem.

20. State and prove open mapping theorem.

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D–4482

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define multiple root and simple root of $P_n(x) = 0$.

2. Define the order of the iteration method.

3. State the Descarte's rule of sign.

4. Define eigen vectors.

5. Write the Newton's interpolating polynomial formula.

6. State the Weierstrass approximation theorem.

7. What is meant by optimal value?

8. Write down the Euler's Back-ward formula.

9. Define grid points.

10. Explain the difference equations.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) By using Regula-Falsi method, determine the root of $\cos x - x e^{x} = 0$.

Or

(b) Perform two iterations of the Bairstow method to extract a quadratic $x^2 + px + q$ from the polynomial $p_3(x) = x^3 + x^2 - x + 2 = 0$.

12. (a) Find the values of a for which the matrix $A = \begin{bmatrix} 1 & a & a \\ -a & 1 & a \\ a & a & 1 \end{bmatrix}$ are positive definite.

Or

(b) Find the interval which contains the eigen values of the symmetric matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$.

13. (a) From the data :

x :	0	1	2	3
$f(x)$:	1	2	33	244

Fit quadratic splines with $M(0) = f''(0) = 0$ and hence find an estimate of $f(2.5)$.

Or

(b) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on $[0, 1]$ using least square approximation with $w(x) = 1$.

14. (a) Consider the four point formula :

$$f'(x_2)=\frac{1}{6h}[-2f(x_1)-3f(x_2)+6f(x_3)-f(x_4)]+$$

$TE+R.E$ where $x_j=x_0+jh,\ j=1,2,3,4$ and TE, RE are respectively the truncation error and round off error. Determine the form of TE and RE.

Or

(b) Use the Euler method to solve numerically the initial value problem $u'=-2tu^2,\ 4(0)=1$ with $h=0.1$ on $[0,1]$.

15. (a) Discretize $y''=t+y,\ y(1)=0$ using backward Euler method and compute $y(1.2)$ using $h=0.1$.

Or

(b) Find the three term Taylor's series solution for the third order initial value problem $W'''+WW''=0$, $W(0)=0, W'(0)=0, W''(0)=1$. Find the bound on the error for $t\in[0,\ 0.2]$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Use synthetic division and perform two interations of the Birge – Vieta method to find the smallest positive root of the polynomial $P_3(x)=2x^3-5x+1=0$. Use the initial approximation $P_0=0.5$.

17. Find all the eigen values of $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.0005.

18. Obtain a least squares fit of the form $f = ae^{-3t} + be^{-2t}$ from the following data :

t :	0.1	0.2	0.3	0.4
f :	0.76	0.58	0.44	0.35

19. Use Taylor's series method of order four to solve $u' = t^2 + u^2, u(0) = 1$ for the interval $(0, 0.4)$ using two subintervals of length 0.2.

20. Use the classical Runge-kutta method of fourth order to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x+y}, y(0.4) = 0.41$. Assume the step length $h = 0.2$.

D–4483

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Sample Space.

2. If the sample space is $\mathcal{C} = C_1 \cup C_2$ and if $P(C_1) = 0.8$ and $P(C_2) = 0.5$, then find $P(C_1 \cap C_2)$.

3. Define distribution function of X and Y.

4. What is meant by conditional probability?

5. Let $f(x, y) = \begin{cases} 2, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

   Show that the correlation coefficient of X and Y is $\rho = \frac{1}{2}$.

6. Define gamma distribution.

7. The moment generating function of the random variable X is $\left(\frac{1}{3} + \frac{2}{3}e^{t}\right)^5$, find $P_r(X = 2 \text{ or } 3)$.

8. Define the concept of "Convergence in distribution".

9. When we say that a sequence of random variables converges in distribution to a random variable with distribution function?

10. State the central limit theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $f(x)=\begin{cases} 1/x^2, & 1<x<\infty \\ 0, & \text{elsewhere} \end{cases}$, be the p.d.f. of X where $A_1 = \{x : 1 < x < 2\}$, $A_2 = \{x : 4 < x < 5\}$, find $P(A_1 \cup A_2)$ and $A(A_1 \cap A_2)$.

Or

(b) Let X and Y have the joint p.d.f.

$$f(x, y) = \begin{cases} 2e^{-x-y}, & 0 < x < y, 0 < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Show that X and Y are independent.

12. (a) Determine the constant e so that

$$f(x)= \begin{cases} ex(3-x)^4, & 0 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function.

Or

(b) Prove that :

(i) $E[E(X_2/X_1)] = E(X_2)$ and

(ii) $Var\,[E(X_2/X_1)] \leq Var(X_2)$.

13. (a) Derive the p.d.f of Binomial distribution.

Or

(b) Suppose that X has a Poisson distribution with $\mu = 2$. Show that the p.d.f. of X is

$$f(x) = \begin{cases} \dfrac{2^x e^{-1}}{x!}, & x = 0, 1, 2, ... \\ 0, & \text{elsewhere} \end{cases}$$

Find $\Pr(1 \le x)$.

14. (a) If $E(x) = 17$ and $E(x^2) = 298$, use Chebyshev's inequality to determine.

(i) A lower bound for $\Pr(10 < x < 24)$

(ii) An upper bound for $\Pr(|x - 17| \ge 16)$.

Or

(b) One of the tasks performed by a computer operator is that of testing tapes in order to detect bad records : Ten observations in X are 67 7 35 78 28 74 5 9 37.

(i) Find the order statistics

(ii) Find the medium and 80th percentile of the sample.

15. (a) Derive the p.d.f. of chi-square distribution.

Or

(b) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu_1, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Derive Chebyshev's inequality.

17. Let the random variables X and Y have the joint p.d.f.

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Compute the correlation coefficient of X and Y.

18. Find the moment generating function, mean and variance of the gamma distribution.

19. Let $T = \frac{W}{\sqrt{V/\gamma}}$ where W and V are respectively normal with mean 0 and variance 1, chi = square with γ, show that T^2 has an F-distribution with parameters $\gamma_1 = 1$ and $\gamma_2 = v$.

20. State and prove central limit theorem.

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